Social Cognitive Theory Dynamical Model

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This document provides additional specifications for the model referred to in the article, "Development of a Dynamical Systems Model of Social Cognitive Theory" by the same authors, which has been accepted to *Translational Behavioral Medicine: Practice, Policy and Research*, ISSN: $1869-6716$.

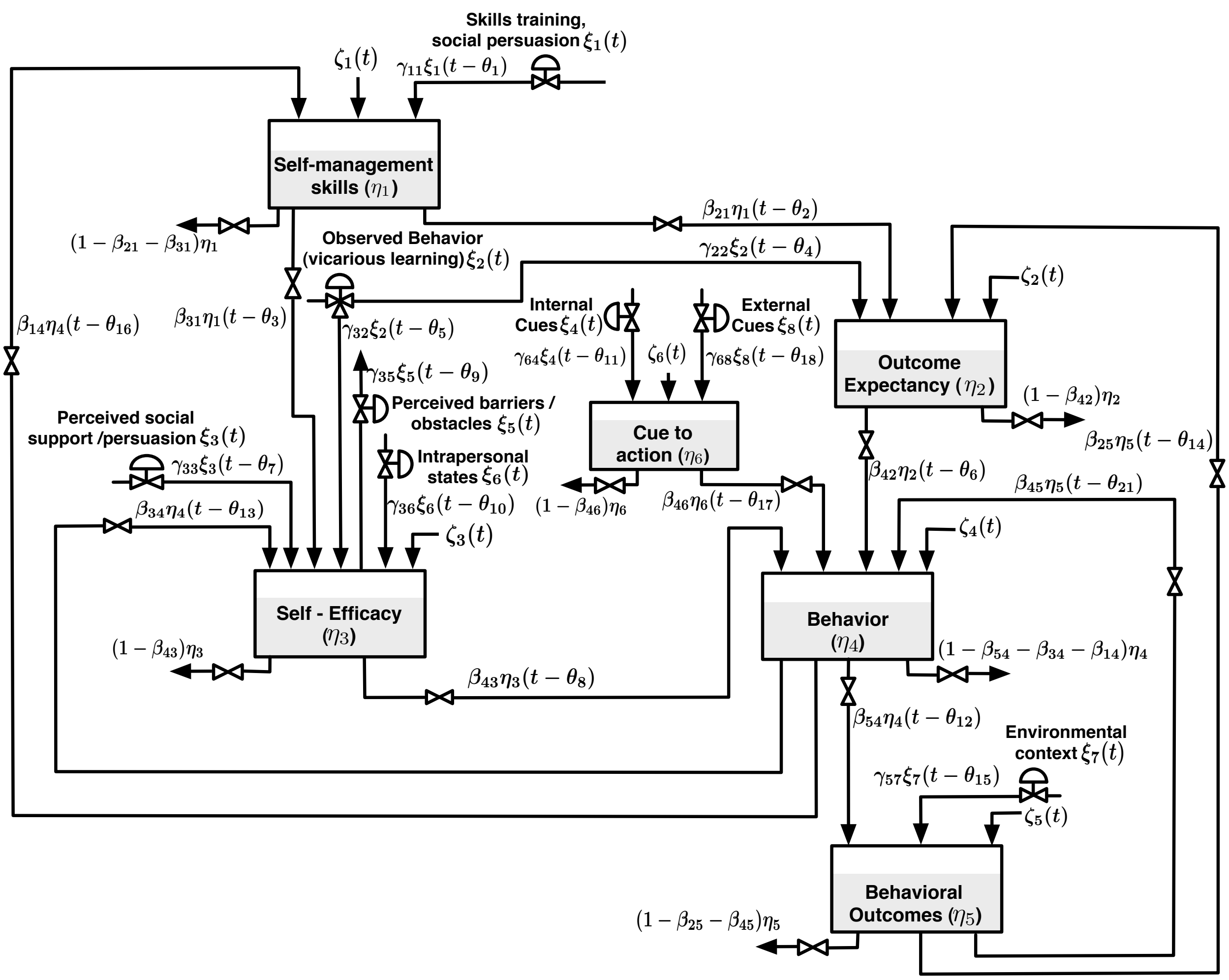

Figure 1: Fluid analogy for Social Cognitive Theory.

1 SCT Model Considerations

Applying the principle of mass conservation to the fluid analogy illustrated in Fig. 1, and assuming first-order dynamics, the following system of differential equations is obtained:

$$\tau_1 \frac{d\eta_1}{dt} = \gamma_{11}\xi_1(t-\theta_1) + \beta_{14}\eta_4(t-\theta_{16}) - \eta_1(t) + \zeta_1(t) \tag{1}$$

$$\tau_2 \frac{d\eta_2}{dt} = \gamma_{22}\xi_2(t-\theta_4) + \beta_{21}\eta_1(t-\theta_2) + \beta_{25}\eta_5(t-\theta_{14}) - \eta_2(t) + \zeta_2(t) \tag{2}$$

$$\begin{aligned}\tau_3 \frac{d\eta_3}{dt} =& \gamma_{32}\xi_2(t-\theta_5) + \gamma_{33}\xi_3(t-\theta_7) - \gamma_{35}\xi_5(t-\theta_9) + \gamma_{36}\xi_6(t-\theta_{10}) \\ &+ \beta_{31}\eta_1(t-\theta_3) + \beta_{34}\eta_4(t-\theta_{13}) - \eta_3(t) + \zeta_3(t)\end{aligned} \tag{3}$$

$$\begin{aligned}\tau_4 \frac{d\eta_4}{dt} =& \beta_{42}\eta_2(t-\theta_6) + \beta_{43}\eta_3(t-\theta_8) + \beta_{46}\eta_6(t-\theta_{17}) + \beta_{45}\eta_5(t-\theta_{19}) \\ &- \eta_4(t) + \zeta_4(t)\end{aligned} \tag{4}$$

$$\tau_5 \frac{d\eta_5}{dt} = \gamma_{57}\xi_7(t-\theta_{15}) + \beta_{54}\eta_4(t-\theta_{12}) - \eta_5(t) + \zeta_5(t) \tag{5}$$

$$\tau_6 \frac{d\eta_6}{dt} = \gamma_{64}\xi_4(t-\theta_{11}) + \gamma_{68}\xi_8(t-\theta_{18}) - \eta_6(t) + \zeta_6(t) \tag{6}$$

Six inventories are considered in the diagram, and their levels are represented by the variables η_1,..,η_6. Eight exogenous inputs are shown and represented by ξ_1,..,ξ_8. From each inventory there are a number of inflow resistances represented by the coefficients γ_{11},..,γ_{68}, and outflow resistances represented by β_{21},..,β_{54}. These resistances can be considered as the fraction of each inventory or input that leaves the previous instance and then feeds the next inventory.

There are other parameters that represent the physical characteristics of each inventory and flow; they have an important effect on the dynamic behavior of the system. First time constants τ_1,..,τ_6 that represent the capacity and allow for exponential decay (or growth) of the inventory, also time delays (θ_1,..,θ_{19}) for each flow signal are used. Unmeasured disturbances (which may reflect unmodeled dynamics) are also considered as ζ_1,..,ζ_6.

For modeling and simulation purposes and to comply with the mass conservation principle, the following assumptions are considered:

- Because the sum of outflows must add up to the value of the respective inventory, the β coefficients must satisfy the following constraints:

$$\beta_{21} + \beta_{31} \leq 1, \quad \beta_{42} \leq 1, \quad \beta_{43} \leq 1$$
$$\beta_{54} + \beta_{34} + \beta_{14} \leq 1, \quad \beta_{25} + \beta_{45} \leq 1, \quad \beta_{46} \leq 1$$

- The initial levels of the inventories are determined by solving the system of equations at steady state, obtaining the following results:

$$
\begin{aligned}
\bar{\eta}_1 &= \gamma_{11}\bar{\xi}_1 + \beta_{14}\bar{\eta}_4 \\
\bar{\eta}_2 &= A + (\beta_{21}\beta_{14} + \beta_{25}\beta_{54})\bar{\eta}_4 \\
\bar{\eta}_3 &= B + (\beta_{31}\beta_{14} + \beta_{34})\bar{\eta}_4 \\
\bar{\eta}_4 &= \frac{\beta_{46}\bar{\eta}_6 + \beta_{42}A + \beta_{43}B + \beta_{45}\gamma_{57}\bar{\xi}_7}{1 - \beta_{42}(\beta_{21}\beta_{14} + \beta_{25}\beta_{54}) - \beta_{43}(\beta_{31}\beta_{14} + \beta_{34}) - \beta_{45}\beta_{54}} \\
\bar{\eta}_5 &= \gamma_{57}\bar{\xi}_7 + \beta_{54}\bar{\eta}_4 \\
\bar{\eta}_6 &= \gamma_{64}\bar{\xi}_4 + \gamma_{68}\bar{\xi}_8
\end{aligned}
$$

where

$$
\begin{aligned}
A &= \gamma_{22}\bar{\xi}_2 + \beta_{21}\gamma_{11}\bar{\xi}_1 + \beta_{25}\gamma_{57}\bar{\xi}_7 \\
B &= \gamma_{32}\bar{\xi}_2 + \gamma_{33}\bar{\xi}_3 - \gamma_{35}\bar{\xi}_5 + \gamma_{36}\bar{\xi}_6 + \beta_{31}\gamma_{11}\bar{\xi}_1
\end{aligned}
$$

- All inventories are restricted to have values within 0 and 100 %.
- The time unit is days and all time delays are considered to be zero.
- Uncertainties are represented as zero mean stochastic signals.
- The exogenous signals: *intrapersonal states* (ξ_6) and *environmental context* (ξ_7) are considered as auto-correlated noise since they may occur in a nearly random way.

Additional information is provided in the ensuing section (next page).

2 Simulation Scenarios

The following simulation scenarios are solved via *Simulink*, a software package part of the MATLAB® family, and using the Dormand-Prince method that is part of the adaptive Runge-Kutta methods for numerical solution of ordinary differential equations. Default solver parameters of the *Simulink* window are used with a relative tolerance of 10^{-3} and an absolute tolerance set to "auto".

Scenarios 1-3 are designed to test the model performance under general conditions and as various inputs are manipulated. The following parameter values are considered:

- $\tau_1 = 1, \tau_2 = 1, \tau_3 = 1, \tau_4 = 2, \tau_5 = 1, \tau_6 = 3$
- $\gamma_{11} = 0.8, \gamma_{22} = 0.75, \gamma_{32} = 1.6, \gamma_{33} = 0.75, \gamma_{35} = 1, \gamma_{36} = 1, \gamma_{57} = 2,$ $\gamma_{64} = 20, \gamma_{68} = 5$
- $\beta_{21} = 0.3, \beta_{31} = 0.5, \beta_{42} = 0.3, \beta_{43} = 0.8, \beta_{45} = 0, \beta_{54} = 0.3, \beta_{34} = 0.2,$ $\beta_{25} = 0.3, \beta_{14} = 0.23, \beta_{46} = 0.435$

Scenarios 4-6 are focused on physical activity interventions and include changes on model parameters to represent different outcomes according to the theory. The considered parameters are:

- $\tau_1 = 1, \tau_2 = 1, \tau_3 = 1, \tau_4 = 2, \tau_5 = 1, \tau_6 = 3$
- $\gamma_{11} = 3, \gamma_{22} = 1, \gamma_{32} = 2, \gamma_{33} = 1, \gamma_{35} = 1, \gamma_{36} = 1, \gamma_{57} = 2,$ $\gamma_{64} = 15, \gamma_{68} = 15$
- $\beta_{21} = 0.3, \beta_{31} = 0.5, \beta_{42} = 0.3, \beta_{43} = 0.8, \beta_{45} = 0.1, \beta_{54} = 0.3, \beta_{34} = 0.2,$ $\beta_{25} = 0.3, \beta_{14} = 0.23, \beta_{46} = 0.44$

Delays (θ_i) and internal disturbance parameters (ζ_i) are considered zero. The random distributions used in the SCT model simulation scenarios are the following:

1. For each simulation scenario unmeasured disturbances (ζ_i) are considered Gaussian with:

$$\zeta_i(k) \sim \mathcal{N}(0, \sigma_i^2) \quad \text{with} \quad \sigma_i^2 = 0.01, \quad i = 1, \cdots, 6$$

2. Inputs *intrapersonal states* (ξ_6) and *environmental context* (ξ_7) are generated as autoregressive signals computed as:

$$\xi_n(k) = \phi_n \xi_n(k-1) + a_n(k), \quad a_n(k) \sim \mathcal{N}(0, \sigma_n^2), \quad n = 6, 7$$

   - for scenario 1: $\phi_6 = 0.6$, $\phi_7 = 0.8$, $\sigma_6^2 = \sigma_7^2 = 100$.
   - for scenarios 2 - 6: $\phi_6 = 0.6$, $\phi_7 = 0.8$, $\sigma_6^2 = 0.25$, $\sigma_7^2 = 2.25$.

Gaussian distributions are commonly assumed to describe random measurement errors, incorporating the definition of finite means and standard deviations. This assumption is a common start point for signal analysis that can be later improved if is required.

2.1 Scenario 1

The first scenario, depicted in Fig. 2, is generated both from theory and empirical data showing that increases in self-efficacy produce increases in behavior. Given partial correlations from SEM studies, both direct and indirect, of approximately 0.5 between self-efficacy and behavior (e.g., 10), we assumed for the simulation that 25% of the variance in behavior should be due to self-efficacy changes. The simulation shown in Fig. 2 shows how changes in social persuasion and observational learning, as one might anticipate would occur from an intervention, results in a change in self-efficacy and a subsequent lagged and attenuated change in behavior that is consistent with 25% of the variance in behavior accounted for by changes in self-efficacy. Additionally, outcome expectancy also increases, albeit modestly, as self-efficacy increases.

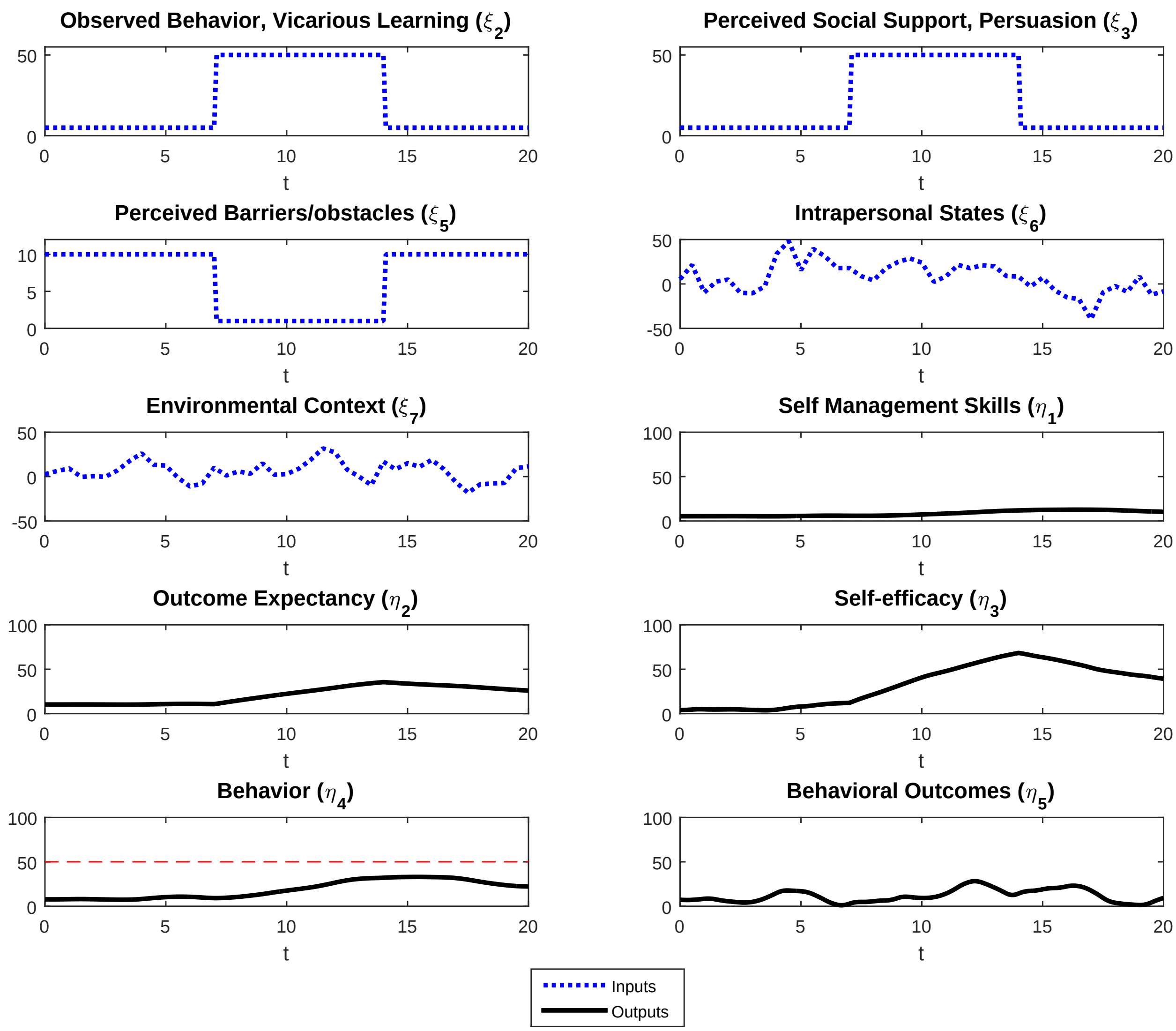

Figure 2: Simulated Scenario of Changing Self-Efficacy Inputs and Effects on Behavior.

2.2 Scenario 2

To simulate low self-efficacy, we held skills training ($\xi_1 = 3$), observed behavior ($\xi_2 = 5$), and perceived social support ($\xi_3 = 5$) at a constant low level. Perceived barriers were maintained at a higher value ($\xi_5 = 10$). Within this context, the input of internal cues are simulated to occur beginning at day 10 ($\xi_4 = 5$) and ending at day 12 (see Fig. 3). The result of the applied signals on the inventories is a small increase of behavior that subsides when the cue to action is subsequently depleted.

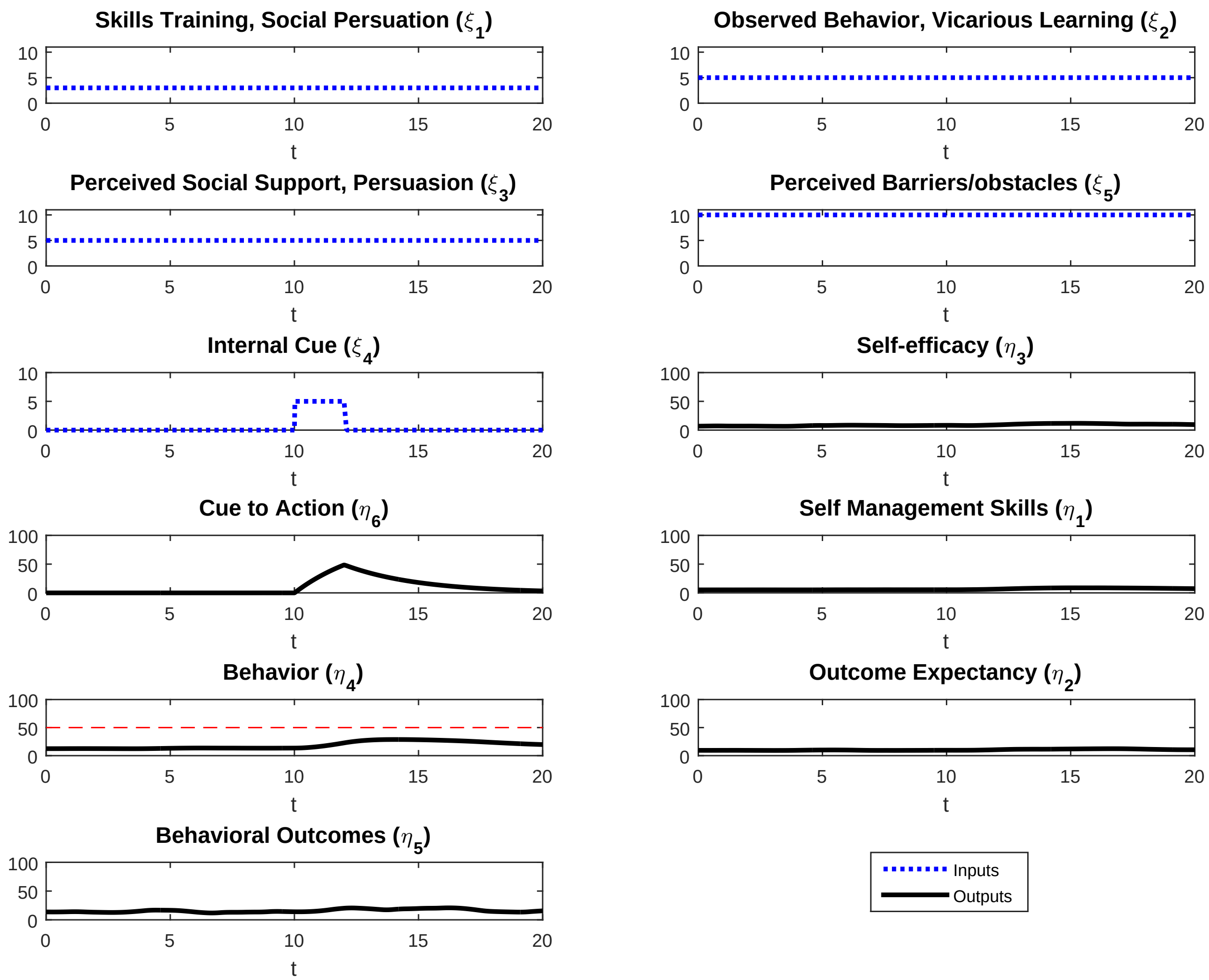

Figure 3: Simulated Scenario of Cue to Action under Low Self-efficacy Condition.

2.3 Scenario 3

Fig. 4 illustrates a similar activity for cue to action at day 10 but within the context of high self-efficacy. Skills training, observed behavior, and perceived social support are kept at high levels ($\xi_1 = 10, \xi_2 = 10, \xi_3 = 10$) and perceived barriers are decreased to a low level ($\xi_5 = 2$). The result is a considerable increase of the behavior inventory. In both scenarios, variations in the other inventories (self-management skills, outcome expectancy and behavioral outcomes) can be observed resulting from the three feedback loops in the system. The simulations represented in Fig. 3 and Fig. 4 represent the role of self-efficacy in attenuating the impact of a cue to action on behavior. The model behaves consistent with theory with weaker effects of cues under conditions of low self-efficacy and stronger effects under conditions of high self-efficacy.

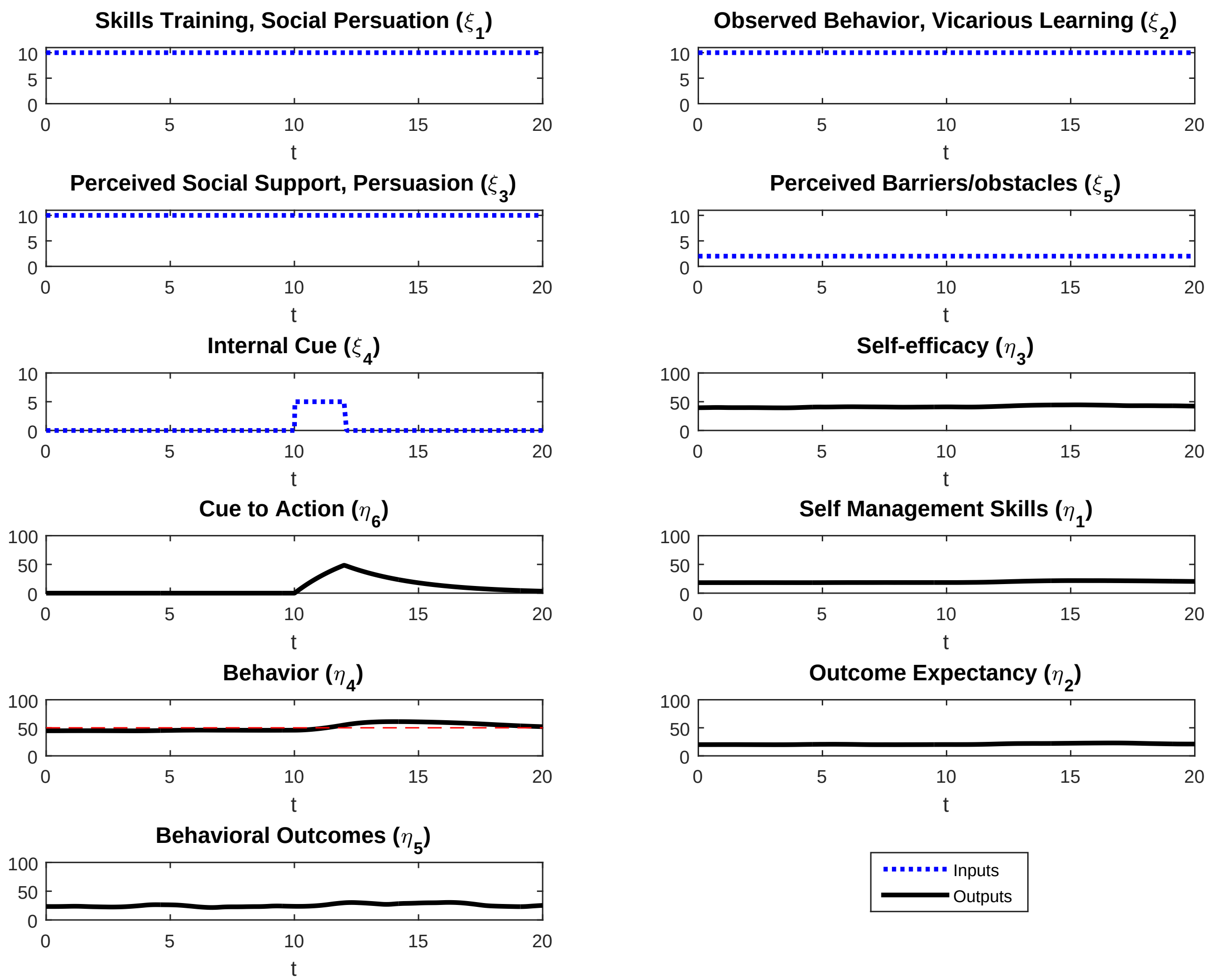

Figure 4: Simulation Scenario of Cue to Action under high Self-efficacy Condition.

2.4 Scenario 4

This scenario is depicted in Fig. 5 and illustrates the effect of external cue to action in the system under conditions of low self-efficacy. To produce low self-efficacy, *observed behavior* ($\xi_2 = 3$), and *perceived social support* ($\xi_3 = 3$) are held at constant low levels. The input *perceived barriers* is kept at a high value ($\xi_5 = 10$). Within this context, an external cue begins on day 2 ($\xi_8 = 5$), subsides on day 6-7, recurs on day 8 and then disappears on day 12. In the physical activity situation these cues could represent calls from friends asking the individual to go on a walk. The result of the applied signals on the inventories is a small, dampened increase on behavior that subsides when the cue to action is subsequently depleted. Because of the low self-efficacy, the engagement on the behavior is minimal.

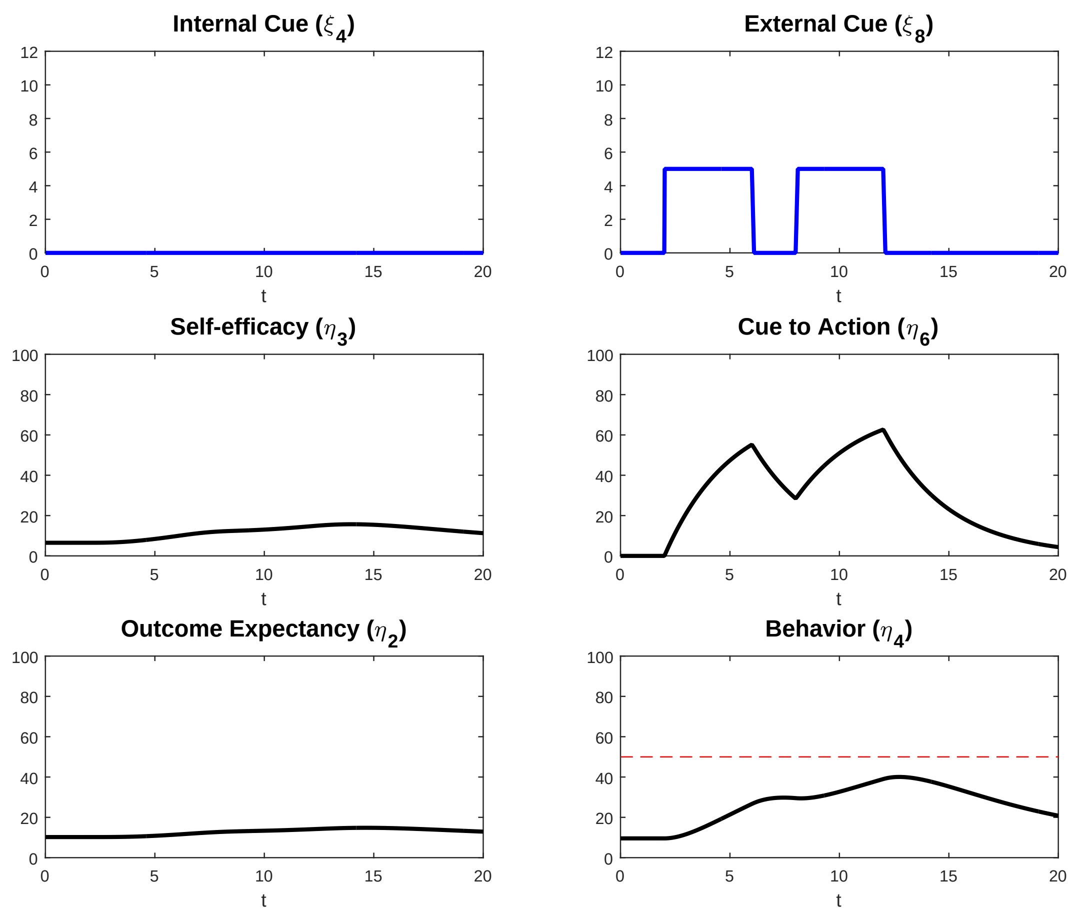

Figure 5: Failure on the initiation of physical activity behavior under low self-efficacy and in the presence of external cues.

2.5 Scenario 5

This scenario, shown in Fig. 6, illustrates an initiation of the behavior and a further maintenance. The *external cue to action* (ξ_8) has similar values as the previous scenario, but within the context of high *self-efficacy*. *Observed behavior* and *perceived social support* are kept at high levels ($\xi_2 = 10, \xi_3 = 10$) and *perceived barriers* are decreased to a low level ($\xi_5 = 2$). The result is a considerable increase on the behavior inventory, and as a consequence some internal cues are now present (example: after a few days walking daily at the same time and one day resting, the individual experiences the internal necessity to walk). The behavior is sustained with fewer cues, but with an increased *self-efficacy* as a result of the feedback loop between SE and behavior.

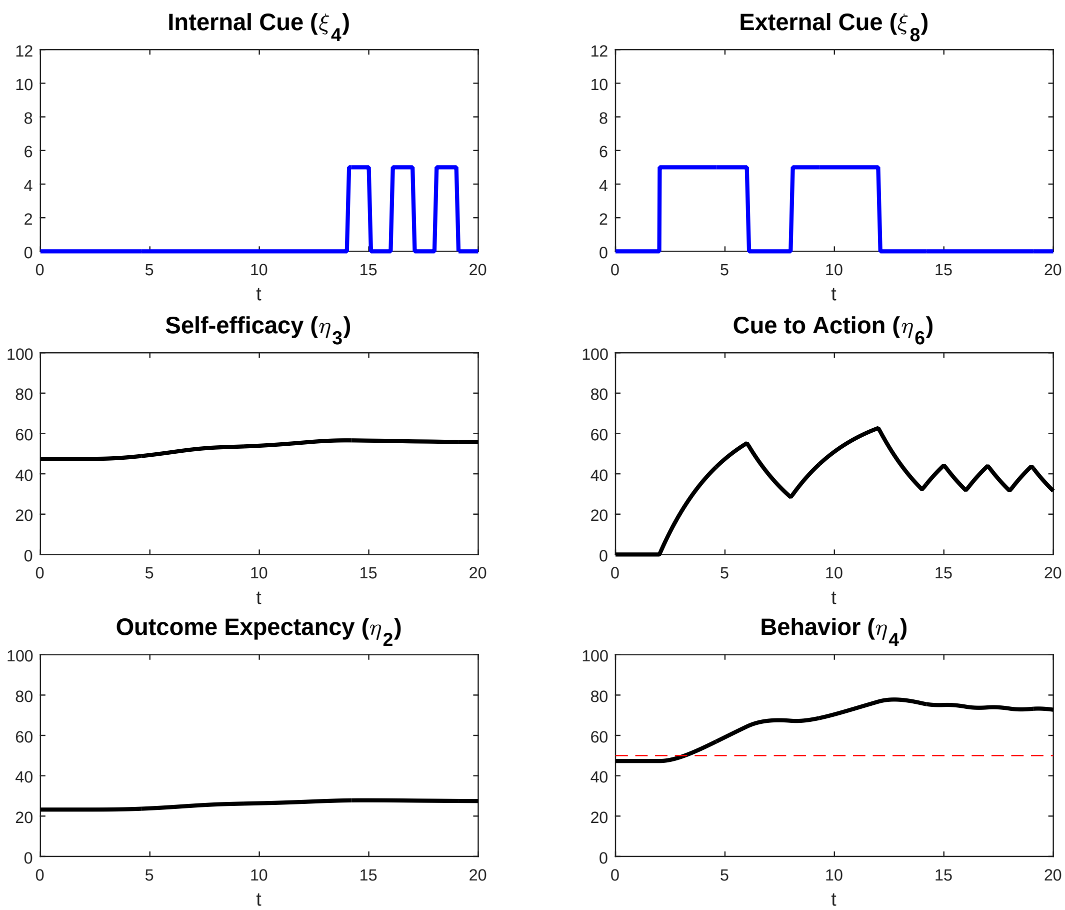

Figure 6: Success on the initiation and maintenance of physical activity behavior under high self-efficacy and in the presence of external and further internal cues.

2.6 Scenario 6

In this scenario another way to represent the maintenance on the magnitude of the behavior is presented. Input conditions are the same as the previous scenario with a high self-efficacy initial condition, but now the recycle loop conformed by the inventories *behavior* (η_4), *behavioral outcomes* (η_5) and *outcome expectancy* (η_2) can be modeled with a more integrative effect so that it is capable of sustaining the magnitude of η_4. This is done with $\tau_4 = 15$ and $\beta_{46} = 0.9$, and results are shown in Fig. 7.

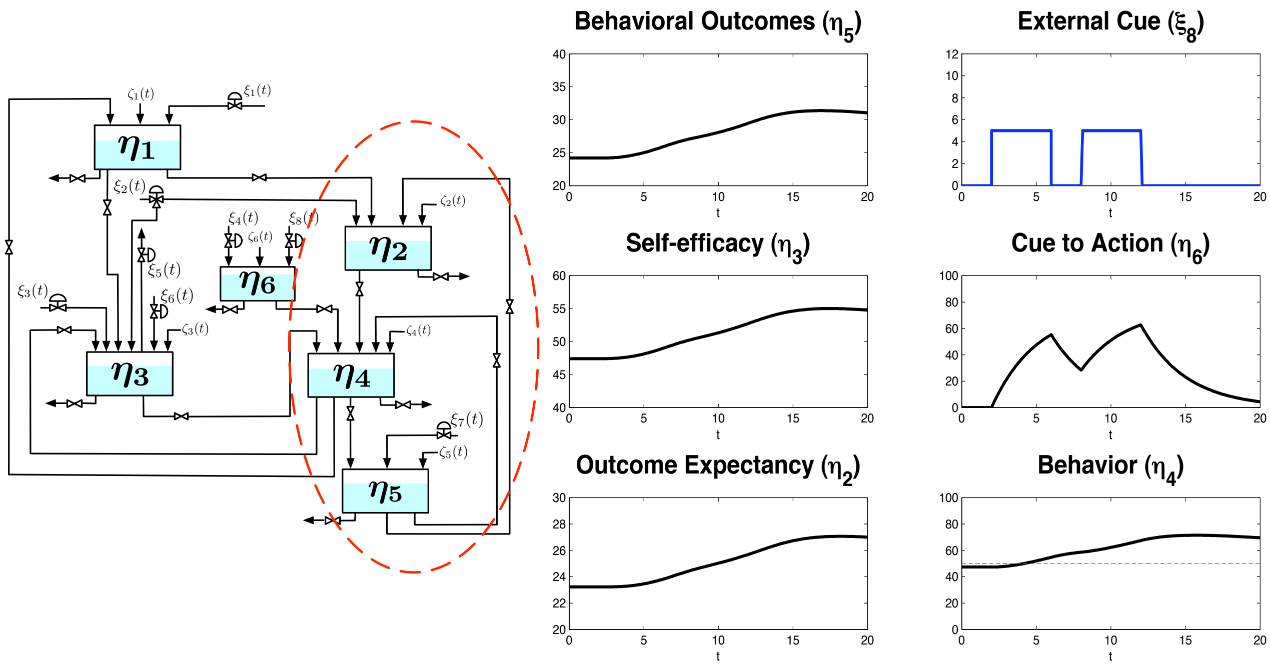

Figure 7: Maintenance of physical activity behavior under high self-efficacy and a model depicting a higher degree of integration.