Identification of Chemical Process Systems Using Constrained Minimum Crest Factor Multisine Inputs

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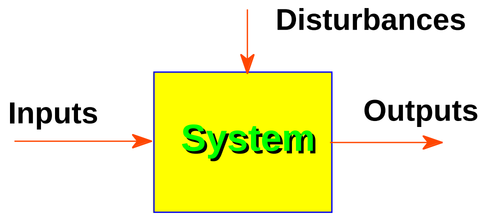
Presentation Outline

- What constitutes "plant-friendly" system identification?
- Minimum Crest Factor Multisine Signal Design
  - Schroeder (1970) phases
  - Guillaume *et al.* (1991) algorithm
  - Constrained problem formulation and solution
- Problem 1: Single-input, single-output (SISO) problem meaningful to control-relevant identification.
- Problem 2: SISO problem meaningful to nonlinear system identification
- Problem 3: Multi-input, multi-output (MIMO) problem, using a highly interactive 2x2 plant example.
- Summary and Conclusions

System Identification

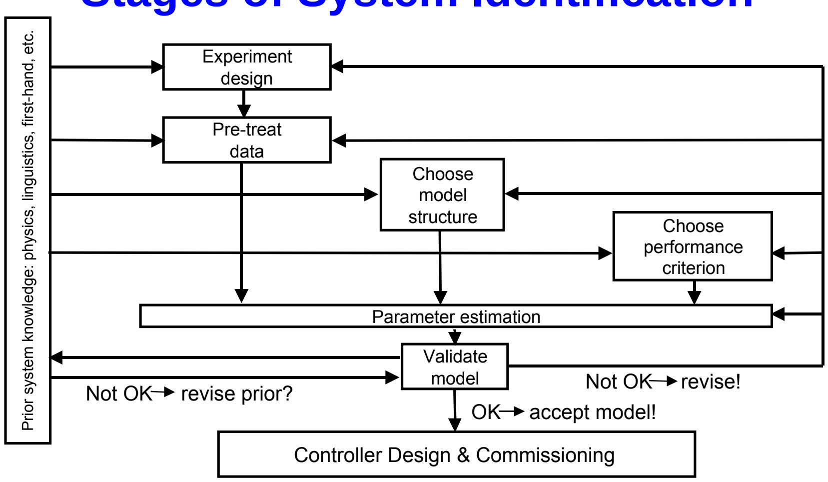
"Identification is the determination, on the basis of input and output, of a system within a specified class of systems, to which the system under test is equivalent."

- L. Zadeh, (1962)



System identification focuses on the modeling of dynamical systems from experimental data

Stages of System Identification



- from P. Lindskog, ISY, Linköping University, Sweden

"Plant Friendly" Input Signal Design

A plant friendly input signal should:

- be as short as possible
- not take actuators to limits, or exceed move size restrictions
- cause minimum disruption to the controlled variables (i.e., low variance, small deviations from setpoint)

Theoretical requirements, however, strongly conflict with "plant-friendly" operation!

Multsine Input Signals

A multisine input is a deterministic, periodic signal composed of a harmonically related sum of sinusoids,

$$u_s(k) = \lambda \sum_{i=1}^{n_s} \sqrt{2\alpha_i}\cos(\omega_i kT + \phi_i)$$

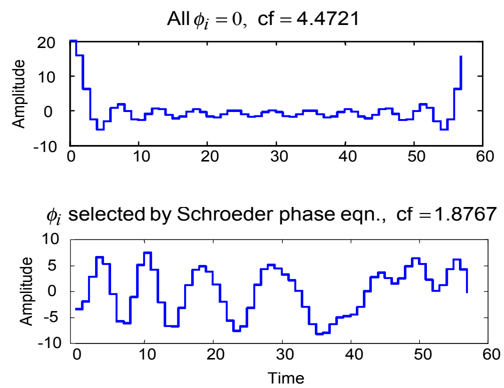
$$\begin{aligned}
T &\equiv \text{sampling time} \\
N_s &\equiv \text{sequence length} \\
n_s &\equiv \text{no. of sinusoids, } n_s \leq N_s/2 \\
\phi_i &\equiv \text{phase angle for harmonic } i \\
\alpha_i &\equiv \text{relative power } \left(\sum_{i=1}^{n_s} \alpha_i = 1\right) \\
\omega_i &= 2\pi i/N_s T \\
\lambda &\equiv \text{scaling factor to insure signal meets } \pm u_{sat}
\end{aligned}$$

The choice of phases ϕ_i, $i = 1, \cdots, n_s$ strongly influences the time-domain realization of the signal.

The Crest Factor (CF) of a signal u is the ratio of the ℓ_∞ (or Chebyshev) norm and the ℓ_2-norm

$$CF(u) = \frac{\ell_\infty(u)}{\ell_2(u)}$$

A low crest factor indicates that most of the elements in the input sequence are distributed near the minimum and maximum values of the sequence.



Schroeder Phases

The work of Schroeder (1970) (Schroeder, M.R. (*IEEE Trans. Information Theory*, **16**, 85, Jan. 1970) presents a closed-form formula to select the phases in a multisine signal

$$\phi_i = 2\pi \sum_{j=1}^{i} j\alpha_j$$

The formula gives a good result when the user-defined spectrum is flat and wideband, but under other conditions (bandlimited, in the presence of harmonic suppression, etc.) the results can be very undesirable.

Research Goal

- Develop a design methodology for multisine inputs that incorporates *both* time-domain "plant-friendliness" constraints and frequency-domain information-theoretic requirements.
- Information-theoretic requirements include such considerations as:
  - persistence of excitation
  - control-relevance
  - harmonic suppression (to address nonlinearity)
  - orthogonality (for multi-input implementation)

Problems with Crest Factor Minimization

- Nonlinear optimization problem
- Nonsmooth (as a consequence of the $\mathcal{L}$-infinity norm)
- Nonconvex objective function

Problems are worsened as a result of increasing problem dimension

Guillame *et al.* (1991) Algorithm

The method of Guillaume *et al.* (1991)* approximates the minimization of the ℓ_∞ norm by sequentially minimizing the ℓ_p norm for $p = 4, 8, 16, \ldots$. It is based on Pólya's algorithm which states that

$$\lim_{p\to\infty} \mathbf{p}_p = \mathbf{p}_\infty$$

where $\mathbf{p} = [\phi_1 \;\; \phi_2 \;\; \phi_3 \;\; \ldots \;\; \phi_{n_s}]$ is the real-valued phase vector for

$$u(k) = \lambda \sum_{i=1}^{n_s} \sqrt{2\alpha_i}\, cos(\omega_i kT + \phi_i)$$

and $\mathbf{p}_\infty$ is the minimax solution. Since the ℓ_2-norm remains invariant with respect to the phases ϕ_i, this method effectively approximates the minimization of the crest factor (CF).

*Guillaume, P., J. Schoukens, R. Pintelon and I. Kollár (1991). Crest-factor minimization using nonlinear chebyshev approximation methods. *IEEE Trans. on Inst. and Meas.*, **40**(6), 982-989.

Constrained Solution Approach

Our solution approach is similar to Guillaume *et al.* (1991) and is based on Pólya's algorithm.

- The problem is formulated in the modeling language AMPL, which provides exact, automatic differentiation up to second derivatives.
- A more gradual increase of p is performed in comparison to Guillaume *et al.* (1991).

Constrained Solution Approach (Continued)

- The trust region, interior point method developed by Nocedal and co-workers (Byrd, R., M.E. Hribar, and J. Nocedal. "An interior point method for large scale nonlinear programming." *SIAM J. Optim.* 9, pgs 877–900, 1999) was applied in our initial research.

- In this talk, improved results based on the Filter-SQP algorithm developed by Fletcher and co-workers (Fletcher, Leyffer, and Toint, "On the Global Convergence of a Filter-SQP Algorithm," Numerical Analysis Report NA/197, University of Dundee, October 2000) are presented.

Problem 1

Given the multisine signal structure

$$u_s(k) = \lambda \sum_{i=1}^{n_s} \sqrt{2\alpha_i} \cos(\omega_i kT + \phi_i)$$

and a known power spectral density (defined by the Fourier coefficients $\lambda\sqrt{2\alpha_i}$ for n_s spectral lines), solve the optimization problem

$$\min_{[\phi_1 \ \ \phi_2 \ \dots \ \phi_{n_s}]} \mathrm{CF}(u_s)$$

subject to maximum move size constraints on the input,

$$|\Delta u_s(k)| \leq \Delta u^{max} \quad \forall\, k$$

and possibly high/low limits on $u(k)$,

$$u^{min} \leq u_s(k) \leq u^{max} \quad \forall\, k$$

Consider a first-order with deadtime plant model

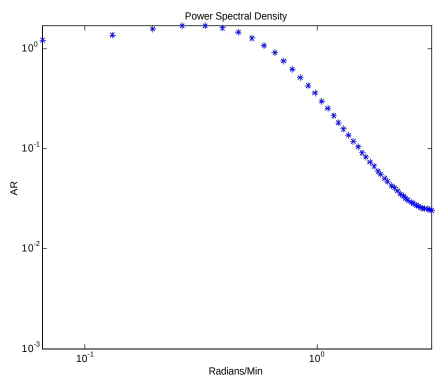
$$\frac{y(s)}{u(s)} = p(s) = \frac{Ke^{-\theta s}}{\tau s + 1} = \frac{e^{-3s}}{3s+1}, \quad T = 1 \text{ min}$$

The power spectrum for the input is chosen based on the control-relevant prefilter result presented in Rivera *et al.* 1992*:

$$L(z) = \tilde{p}_e(z)\tilde{p}^{-1}(z)\tilde{\epsilon}(z)\tilde{\eta}(z)(r(z) - d(z))$$

$$\begin{aligned} \tilde{p}(z) &\equiv \text{plant model} \\ \tilde{p}_e(z) &\equiv \text{noise model} \\ \tilde{\eta}(z) &= \tilde{p}c(1+\tilde{p}c)^{-1} \text{ complementary sensitivity} \\ \tilde{\epsilon}(z) &= (1+\tilde{p}c)^{-1} \text{ sensitivity} \\ r(z) - d(z) &\equiv \text{setpoint/disturbance direction} \end{aligned}$$

*Rivera, D.E., J.F. Pollard, and C.E. García, (1992) "Control-Relevant Prefiltering: A Systematic Design Approach and Case Study," *IEEE Trans. Autom. Cntrl.*, **37**, 964-974.



$$u(z) = (z-\alpha)(1 - z^{-nk}f(z))z^{-1}f(z)(r(z)-d(z)), \qquad f(z) = \frac{(1-\delta)^2 z^2}{(z-\delta)^2}$$

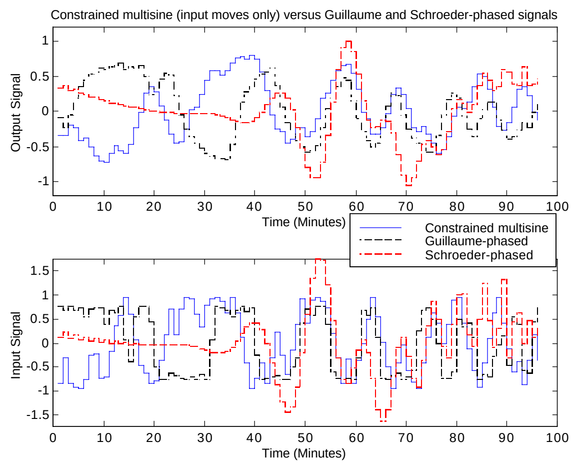
$\delta = e^{-1.555T/\tau_{cl}}$; $\tau_{cl} = 2.25$ min is the desired closed-loop time constant. $\alpha = e^{-T/\tau_{dom}}$; $\tau_{dom} = 4.5$ min is a dominant time constant estimate

Problem 1 Example Results

Signal	CF(u)	max(Δu)	min(u)	max(u)	CF(y)	max(Δy)	min(y)	max(y)
Schroeder-phased	2.7966	1.6214	-1.6384	1.75	2.5892	0.4846	-1.0524	0.9984
Guillaume-phased	1.2173	1.1013	-0.7618	0.7612	1.7020	0.3506	-0.6918	0.6835
min CF(u) Constrained ($\Delta u^{max} = 0.525$)	1.5051	0.525	-0.9417	0.9413	1.9658	0.3487	-0.7280	0.7990
min CF(y) Unconstrained	2.4036	1.3119	-1.3778	1.5052	1.2032	0.4405	-0.4891	0.4861
min CF(y) ($\Delta u^{max} = 0.525$; $\Delta y^{max} = 0.4$)	2.039	0.525	-1.2758	1.2632	1.3860	0.3300	-0.5601	0.5634

- Maximum move size reduced by 50% with only a relatively low (~24% increase) in crest factor and overall input signal span

Constrained min CF vs Guillame and Schroeder-phased signals



Constrained multisine input signal (with $\Delta u^{max} = 0.525$) compared to Guillaume-phased and Schroeder-phased signals.

Problem 1(b)

Minimize the crest factor of the plant *output*

$$\min_{[\phi_1 \ \phi_2 \ \cdots \ \phi_{n_s}]} \mathrm{CF}(y)$$

subject to maximum move size constraints on the input

$$|\Delta u_s(k)| \leq \Delta u^{max} \quad \forall\, k$$

output variability constraints

$$|\Delta y(k)| \leq \Delta y^{max} \quad \forall\, k$$

and high/low limits on $u_s(k)$ and $y(k)$

$$u^{min} \leq u_s(k) \leq u^{max} \quad \forall\, k$$

$$y^{min} \leq y(k) \leq y^{max} \quad \forall\, k$$

This scenario assumes that some *a priori* knowledge of the model is available to the optimizer to generate predicted outputs.

ASU

Problem 1 Example Results

Signal	$\mathrm{CF}(u)$	$\max(\Delta u)$	$\min(u)$	$\max(u)$	$\mathrm{CF}(y)$	$\max(\Delta y)$	$\min(y)$	$\max(y)$
Schroeder-phased	2.7966	1.6214	-1.6384	1.75	2.5892	0.4846	-1.0524	0.9984
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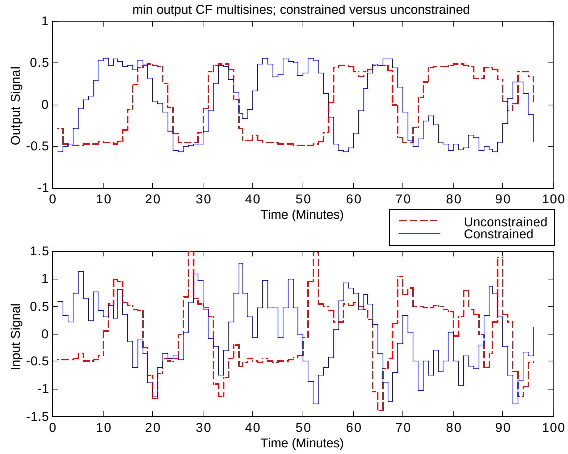
{ min $\mathrm{CF}(y)$ ($\Delta u^{max} = 0.525$; $\Delta y^{max} = 0.4$)	2.039	0.525	-1.2758	1.2632	1.3860	0.3300	-0.5601	0.5634

Constrained problem solution sharply reduces the input move size and output changes. There is an unavoidable increase in the output CF, but the input CF and input/output spans are brought within acceptable levels.

Minimum Output CF signals (constrained vs unconstrained)



Constrained multisine input signal (with $\Delta u^{max} = 0.525$, $\Delta y^{max} = 0.4$) compared to unconstrained multisine result (output crest factor minimized).

Problem 2

Given the multisine signal structure below and a known power spectrum (defined by the Fourier coefficients $\lambda\sqrt{2\alpha_i}$ for n_s spectral lines)

$$u_s(k) = \lambda \sum_{i=1}^{n_s} \sqrt{2\alpha_i} \cos(\omega_i kT + \phi_i) + \sum_{i=n_s+1}^{n_a+n_s} \hat{a}_i \cos(\omega_i kT + \phi_i^a)$$

where $n_a + n_s \leq N_s/2$ and $\hat{a}_i$ and ϕ_i^a represent the Fourier coefficients and phases, respectively, for n_a spectral lines beyond those defined in the user-specified spectrum, solve the optimization problem

$$\min_{[\phi_1 \ \ \phi_2 \ \cdots \ \phi_{n_s}],[\phi^a_{n_s+1} \ \ \phi^a_{n_s+2} \ \cdots \ \phi^a_{n_s+n_a}],[\hat{a}_{n_s+1} \ \ \hat{a}_{n_s+2} \ \cdots \ \hat{a}_{n_s+n_a}]} \mathrm{CF}(u_s)$$

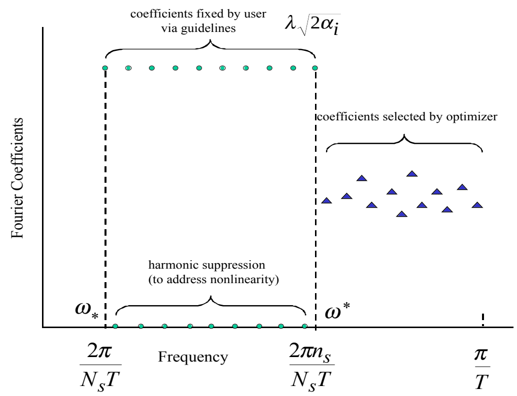
subject to maximum move size constraints on the input,

$$|\Delta u_s(k)| \leq \Delta u^{max} \quad \forall \ k$$

and possibly high/low limits on $u(k)$,

$$u^{min} \leq u_s(k) \leq u^{max} \quad \forall \ k$$

ASU

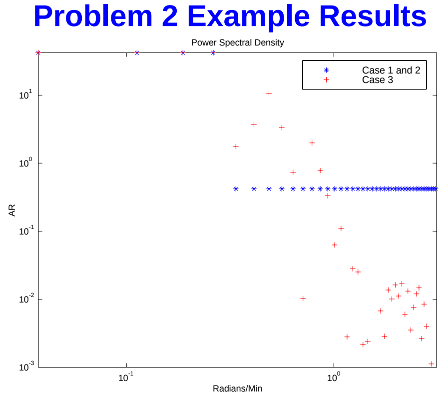


Flat spectrum displaying harmonic suppression with optimized Fourier coefficients. The known part of the spectrum is defined by:

$$\omega_* = \frac{1}{\beta_s \tau_{dom}^H} \leq \omega \leq \frac{\alpha_s}{\tau_{dom}^L} = \omega^*.$$

τ_{dom}^H and τ_{dom}^L correspond to the high and low estimates of the dominant time constant of the system. α_s and β_s correspond to user-decisions on the high and low frequency content of the signal, respectively.

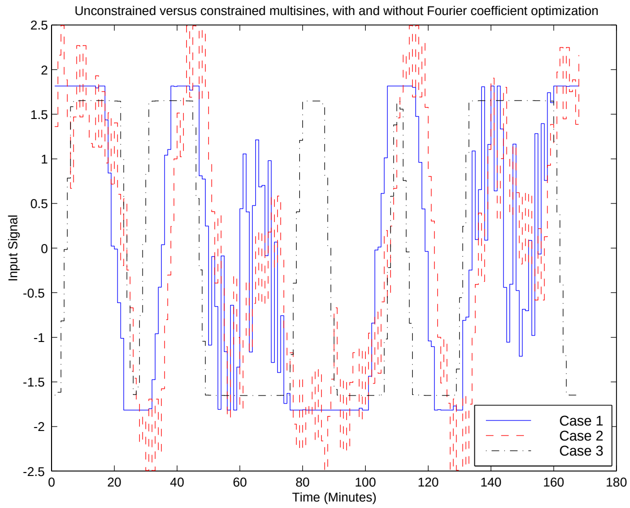
Problem 2 Example Results



Signal	CF(u)	max(Δu)	min(u)	max(u)
Case 1	1.2274	2.2618	-1.8164	1.8164
Case 2	1.6854	0.8	-2.4942	2.4942
Case 3	1.0947	0.8	-1.6532	1.6532

Problem 2 example results; Case 1 (unconstrained), Case 2 ($\Delta u^{max} = 0.8$), Case 3 ($\Delta u^{max} = 0.8$; high-frequency Fourier coefficients optimized).

Problem 2 Example Results (Cont.)



Problem 2 example results; Case 1 (unconstrained), Case 2 ($\Delta u^{max} = 0.8$), Case 3 ($\Delta u^{max} = 0.8$; high-frequency Fourier coefficients optimized).

Problem 3: Highly Interactive Multivariable System

$$P(s) = \frac{1}{75s+1}\begin{bmatrix} 87.8 & -86.4 \\ 108.2 & -109.6 \end{bmatrix}$$

- Highly interactive (i.e., ill-conditioned) plants represent challenging problems for identification and control
- "Model 0" from Morari and Zafiriou, Robust Process Control, Prentice-Hall, (1987) has been studied.
- Simplest *meaningful* highly interactive problem we could find...

Highly Interactive Systems

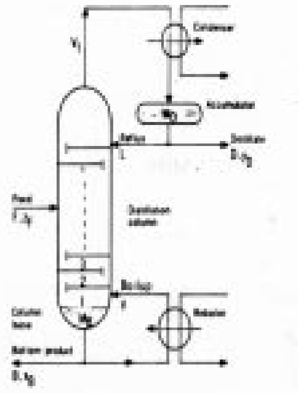


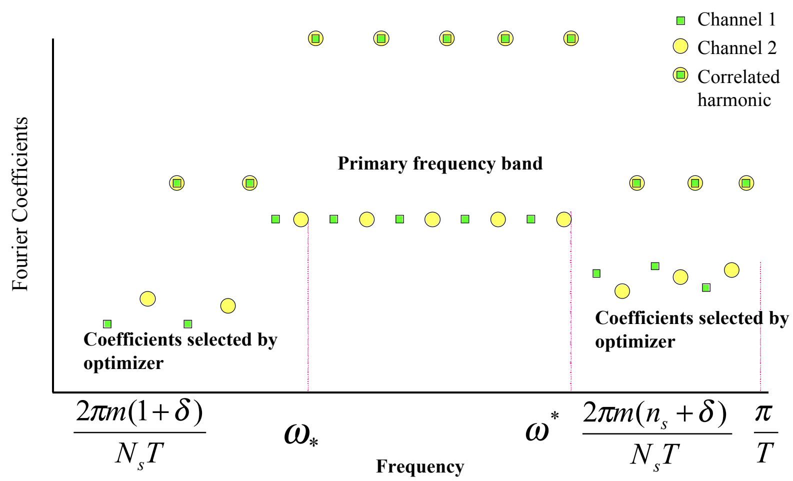
Fig. 1. Two-product distillation column.

Highly interactive plants (such as high purity two-point binary distillation) present some inherent challenges to control design

Constrained Multisine Approaches for Highly Interactive Systems

- Open-loop design using a modified "zippered" spectrum. Overlapping/correlated sinusoids are introduced in the signal spectrum; these emphasize low-gain directionality in the data.
- Closed-loop design: Multisine signals are introduced as reference setpoints to the process under some form of closed-loop control.

Open-Loop Design using a Modified "Zippered" Spectrum



Open-Loop Signals Summary

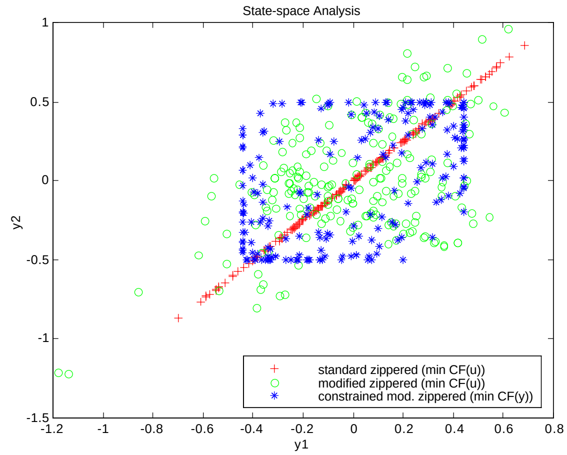
Type	Signal (x)	CF(x)	PIPS(%)	max Δx	max x	min x
min CF (u) design; standard zippered spectrum	u_1	1.132425	88.306039	0.014106	0.007079	-0.007079
	u_2	1.136376	87.999046	0.014201	0.007100	-0.007100
	y_1	2.383208	42.520481	0.318468	0.681899	-0.700354
	y_2	2.372958	42.530317	0.397652	0.855283	-0.871213
min CF (u) design; modified zippered spectrum	u_1	1.130450	88.460763	0.745004	0.372522	-0.372526
	u_2	1.130632	88.446404	0.745163	0.372585	-0.372583
	y_1	3.924442	33.358140	0.763222	0.619802	-1.174440
	y_2	3.459978	32.254284	0.863657	0.966114	-1.219642
min CF (y) design; modified zippered, $(\Delta u, \Delta y, y)^{max} \leq 0.5$, $y^{min} > -0.5$.	u_1	1.423676	70.240749	0.499997	0.464209	-0.464209
	u_2	1.423675	70.240777	0.499995	0.464217	-0.464217
	y_1	1.423637	70.242767	0.407867	0.441263	-0.441262
	y_2	1.363417	73.346052	0.499934	0.499980	-0.499967

Performance Index for Perturbation Signals (PIPS):

$$PIPS(\%) = 200\frac{\sqrt{x_{rms}^2 - x_{mean}^2}}{x_{max} - x_{min}}$$

State-space Comparison

standard (+), modified unconstrained (o) and modified with constraints (*) zippered designs



Summary and Conclusions

- A constrained problem formulation allowing the user to specify desirable frequency-domain and time-domain specifications in low crest factor multisines has been presented.
- Its usefulness has been demonstrated in a series of problems ranging from signals for control-relevant identification to input signal design for highly interactive multivariable systems.
- ***The effective use of a priori knowledge is critical in the solution of this (or any other) control-oriented, plant-friendly identification problem***.

Acknowledgements

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References

- Rivera, D.E., M.W. Braun, and H.D. Mittelmann, "Constrained multisine inputs for plant-friendly identification of chemical processes," 15th IFAC World Congress, Barcelona, Spain, July 21-26, 2002.
- Rivera, D.E., H. Lee, M.W. Braun, and H.D. Mittelmann, "Identification of multivariable process systems using constrained minimum crest factor multisine inputs," Paper 255f, 2002 AIChE Annual Meeting, Indianapolis, IN, Nov. 3-8, 2002.

  Preprints of both papers can be downloaded from the CSEL website, http://www.eas.asu.edu/~csel/pubs.htm)