ECE 100: Introduction to Engineering Design

Modeling Assignment No. 3
Inventory Management Using PID Control (Continued); Stochastic Modeling

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Some Course Business Items…

- Submit your Modeling Assignment No. 3 to your team folder.

Operational Objectives of the Inventory Management Control System

- Setpoint Tracking.
- Disturbance Rejection.

Setpoint Tracking

- Refers to the ability of the control system to manipulate orders such that the controlled variable (net stock or inventory position) follows a reference (setpoint) trajectory as closely as possible.

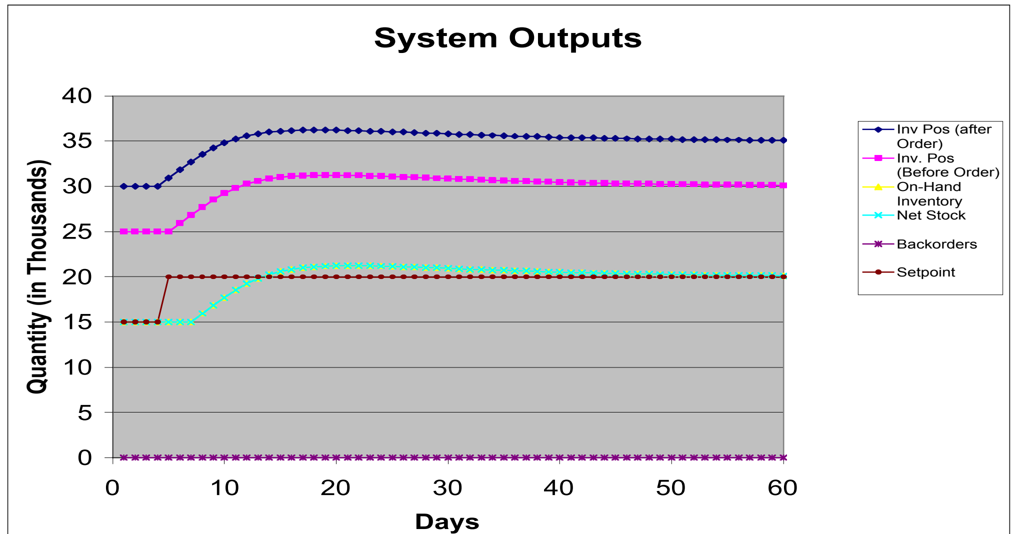
- Changing the inventory position/net stock setpoint from say, 20K units to 10K units while leaving demand unchanged will allow you to observe the *setpoint tracking* ability of your control system.

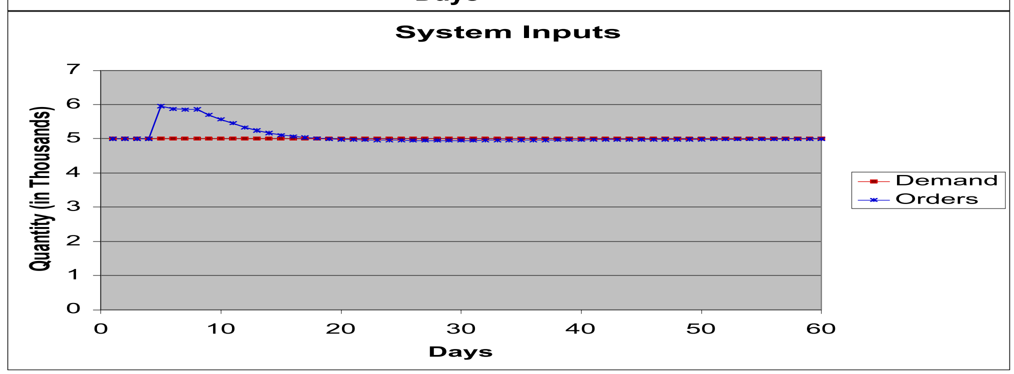
Setpoint Tracking Example

Net Stock as Controlled Variable

(Take Net Stock from 10K units/day to 15K units/day at day = 5)

IMC-PID Decision Policy - Net Stock Controlled Variable	
DE Rivera (Team INSTRUCTOR)	
Initial Net Stock (in K units)	15
Initial Inventory Position (Calculated)	25
Initial (Baseline) Demand (in K units)	5
Initial Orders (O(-2) = O(-1) = O(0))	5
Order Fulfillment Time (Theta)	3
Controller Tuning Parameter (Lambda)	10
Beta	1.5
Tau	1.5
Sampling Time (Ts)	1
Setpoint Change (at k = 5 days)	5
Proportional Gain (Kc)	0.14893617
Integral Time (tauI)	24.5
Derivative Time (tauD)	1.408163265
Filter Time Constant (tauF)	0.911854103
Coeff Kf1	0.190779014
Coeff Kf2	-0.297297297
Coeff Kf3	0.109697933
Coeff Kf4	0.476947536
Inventory Holding Cost ($/K units):	$100
Order Cost ($/order):	$100
Backorder Cost ($/K units):	$1,000
Average On-Hand Inventory (in K units)	19.7
Average Backorders (in K units)	0
Total Orders	60
Total Order Cost	$6,000
Total Inventory Holding Costs	$118,322.30
Total Backorder Costs	$0.00
Total Cost	$124,322.30
Norm Criteria	
RMS Error	1.64
Max Error	5.00





Disturbance Rejection

- Refers to the ability of the control system to manipulate orders such that the controlled variable is kept as close as possible to the setpoint, despite changes in demand.

- Keep in mind that the demand changes to be evaluated in your spreadsheet model will involve the sum of deterministic and stochastic (random) components.

  Demand Change (or Demand Variation) = Random + Deterministic

  Total Demand = Nominal (Baseline) Demand + Demand Change

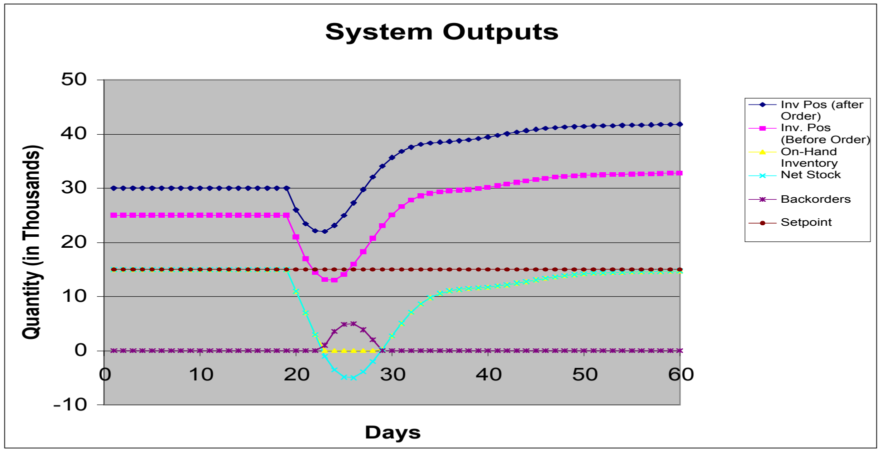
- Demand change generation should reside in a separate worksheet (see example in SCMstart2003ver2.xls).

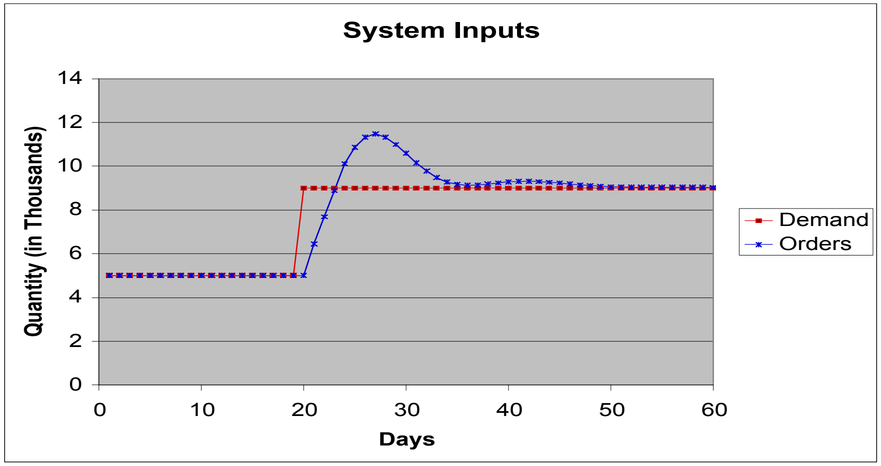
- Incorporating the stochastic (random) component will be described later in the presentation.

Deterministic Disturbance Rejection Example

Net Stock as Controlled Variable
(+4k/day demand increase at day 20)

IMC-PID Decision Policy - Net Stock Controlled Variable	
DE Rivera (Team INSTRUCTOR)	
Initial Net Stock (in K units)	15
Initial Inventory Position (Calculated)	25
Initial (Baseline) Demand (in K units)	5
Initial Orders (O(-2) = O(-1) = O(0))	5
Order Fulfillment Time (Theta)	3
Controller Tuning Parameter (Lambda)	5
Beta	1.5
Tau	1.5
Sampling Time (Ts)	1
Setpoint Change (at k = 5 days)	0
Proportional Gain (Kc)	0.243697479
Integral Time (tauI)	14.5
Derivative Time (tauD)	1.344827586
Filter Time Constant (tauF)	0.630252101
Coeff Kf1	0.360824742
Coeff Kf2	-0.551546392
Coeff Kf3	0.201030928
Coeff Kf4	0.386597938
Inventory Holding Cost ($/K units):	$100
Order Cost ($/order):	$100
Backorder Cost ($/K units):	$1,000
Average On-Hand Inventory (in K units)	11.4
Average Backorders (in K units)	0.338760121
Total Orders	60
Total Order Cost	$6,000
Total Inventory Holding Costs	$68,428.28
Total Backorder Costs	$20,325.61
Total Cost	$94,753.89
Norm Criteria	
RMS Error	7.05
Max Error	19.99





Assessing Closed-Loop Performance

- Deterministic Measures
  - Bounded Input, Bounded Output (BIBO) stability. Bounded changes in demand result in bounded changes in orders and inventories. In a stable response, there is *convergence* to a steady-state, as opposed to *divergence*.
  - Response characteristics.
    - Shape of response (i.e., smooth or oscillatory)
    - Offset (control error does not go to zero after long enough time)
    - Settling time
    - Overshoot/undershoot

*Please examine the **clperformance.pdf** handout (posted on the course website) for more details*

Assessing Closed-Loop Performance (Continued)

- Deterministic Measures (*Norm* Criteria)

The Root-Mean-Square (RMS) control error is computed as

$$RMSerr = \left(\frac{1}{N} \sum_{k=1}^{N} e^2(k) \right)^{1/2} = \left(\frac{1}{N} \sum_{k=1}^{N} (r(k) - y(k))^2 \right)^{1/2}$$

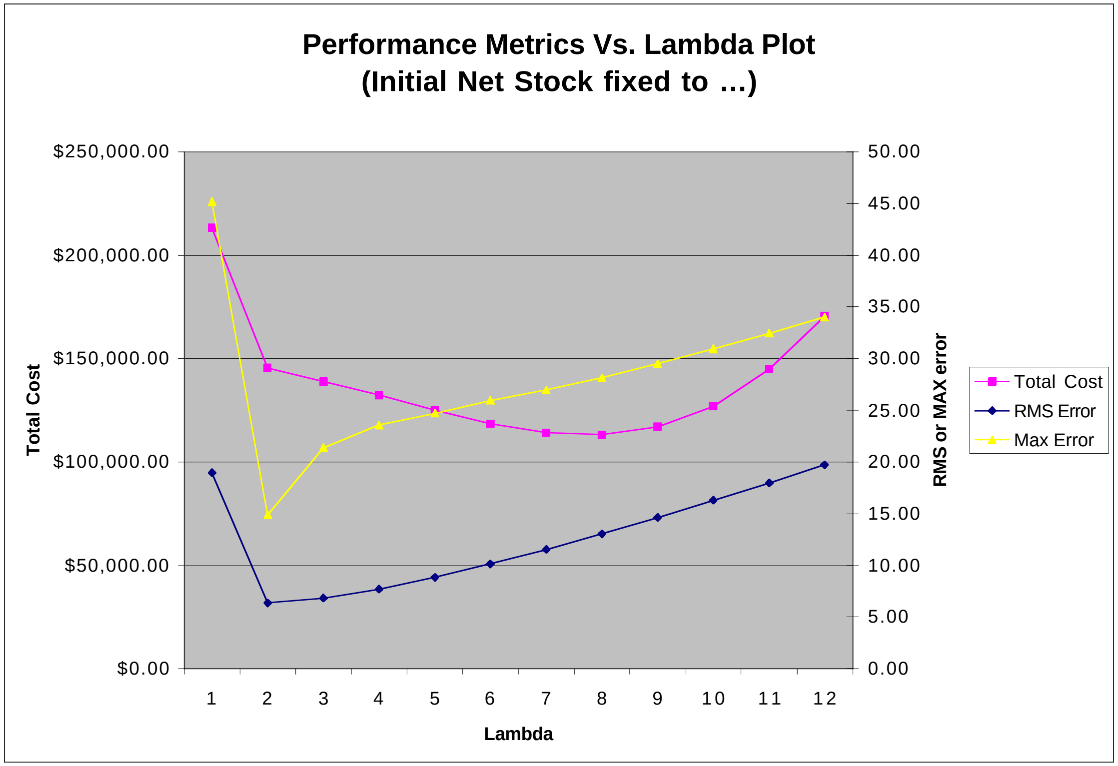
while the maximum (MAX) control error consists of the largest absolute magnitude error

$$MAXerr = \max_k |e(k)| \qquad k = 1, \cdots, N$$

N is the total number of days in the simulation run.

Use SQRT, SUM, MAX, and ABS commands to implement these measures in your spreadsheet! You will need to create new columns to compute these properly.

Using Deterministic Norm Criteria to Determine the Best Choice of Controlled Variable



Conditions kept ambiguous (on purpose!)
Note the use of a *secondary* axis on chart

Stochastic Modeling

- A *stochastic* component will be introduced into our simulation by considering the effects of random changes in demand on the behavior of the "closed-loop" inventory management system
- RAND() returns an evenly distributed random number greater than or equal to 0 and less than 1.
- F9 key causes a recalculation of RAND and hence generates a new *realization* of the random process
- Random demand generation formulas are posted in SCMstart2003ver2.xls
- Setting the *Gain* parameter in the demand page will influence the magnitude of the random noise on the simulation.

Assessing Closed-Loop Performance

- Stochastic Measures
  - Mean (calculated via the AVERAGE function)
  - Variance (calculated via the VAR function)

These can be computed on any time-varying quantity in the system (control error, orders, demand, etc.)

In general, a system is experiencing the "bullwhip" effect if the variance of the orders is much larger than that of the incoming demand.

Assessing Closed-Loop Performance (Continued)

The *mean* of a signal x ($\bar{x}$) is computed as

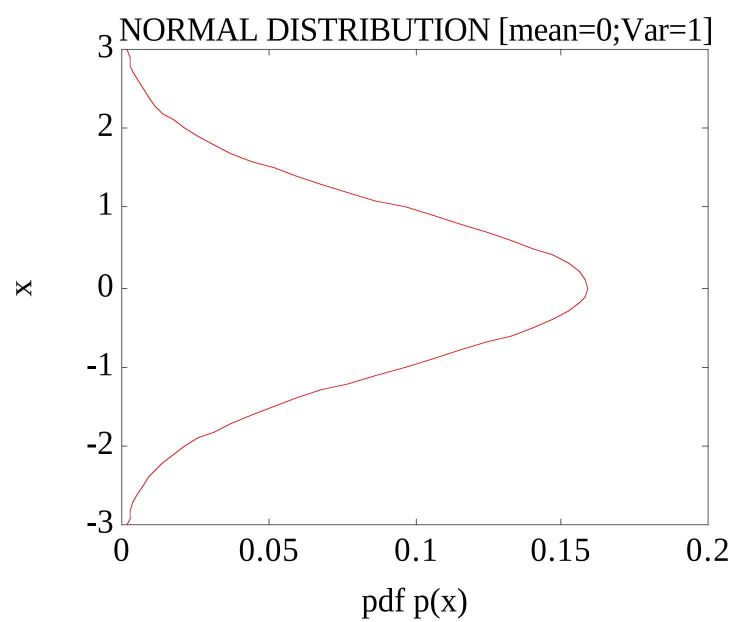
$$\bar{x} = \frac{1}{N}\sum_{k=1}^{N} x(k)$$

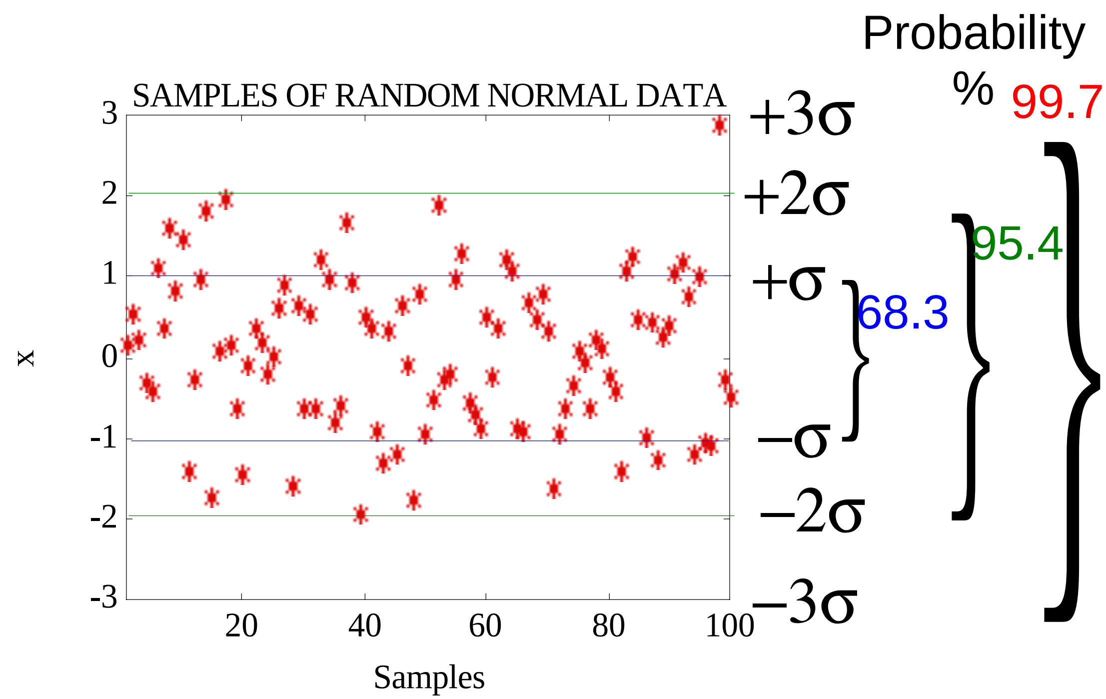
while the *variance* of the signal is determined from

$$\mathrm{var}[x] = \left(\frac{1}{N}\sum_{k=1}^{N}(x(k) - \bar{x})^2\right)^{1/2}$$

N is the total number of days in the simulation run.

A Normally Distributed (Gaussian) Random Distribution





σ^2 is the variance of a normally distributed variable

σ is one standard deviation

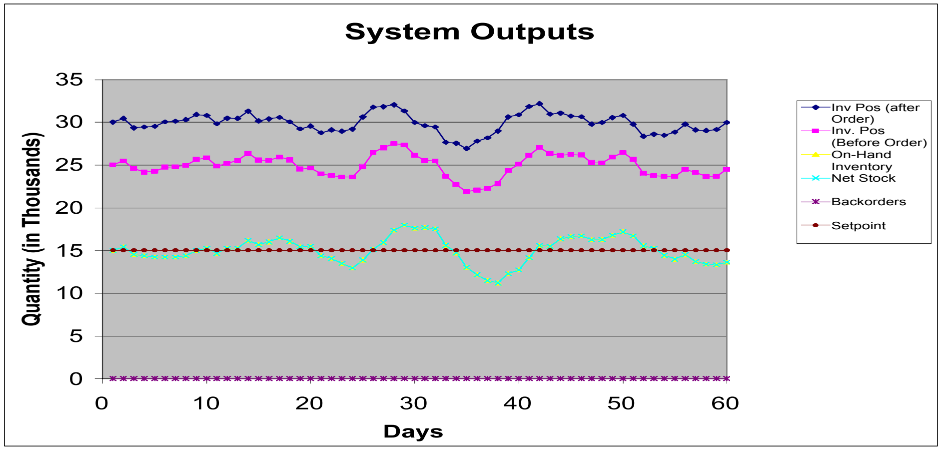
Stochastic (Random) Disturbance Rejection Example - Net Stock as Controlled Variable

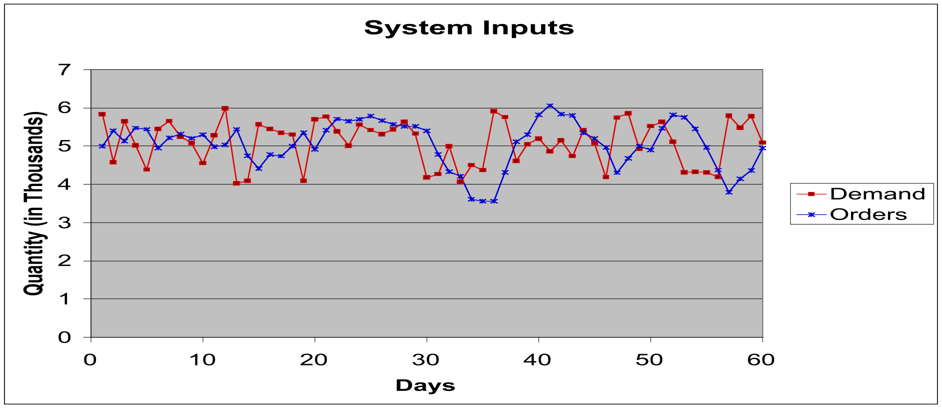
Note: you will not be able to exactly reproduce these results

Initial Net Stock (in K units)	15
Initial Inventory Position (Calculated)	25
Initial (Baseline) Demand (in K units)	5
Initial Orders (O(-2) = O(-1) = O(0))	5
Order Fulfillment Time (Theta)	3
Controller Tuning Parameter (Lambda)	3.5

Variability Analysis	
Control Error Mean	-0.079565647
Control Error Variance	2.862398481
Order Mean	5.073971379
Order Variance	0.378958061
Demand Mean	5.105986992
Demand Variance	0.353088144

Stochastic Gain = 2;
NO Setpoint Change
NO Deterministic Disturbance Change





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Stochastic + Deterministic Disturbance Rejection Example - Net Stock as Controlled Variable

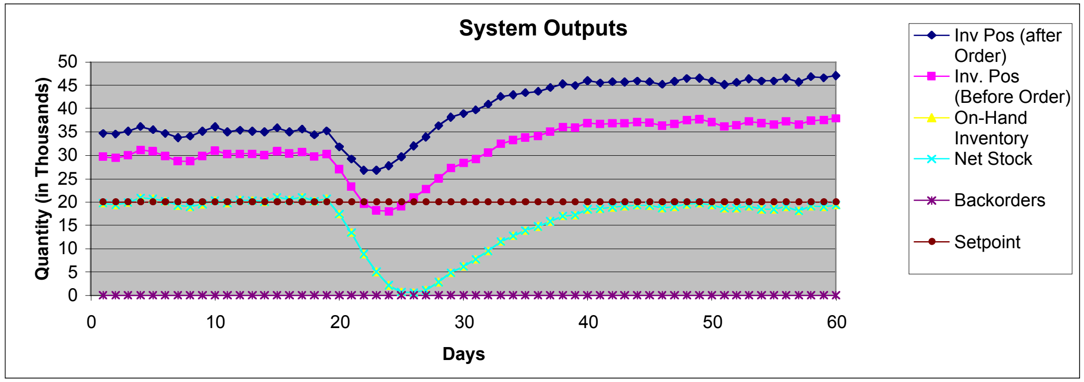
Initial Net Stock (in K units)	20
Initial Inventory Position (Calculated)	30
Initial (Baseline) Demand (in K units)	5
Initial Orders (O(-2) = O(-1) = O(0))	5
Order Fulfillment Time (Theta)	3
Controller Tuning Parameter (Lambda)	5

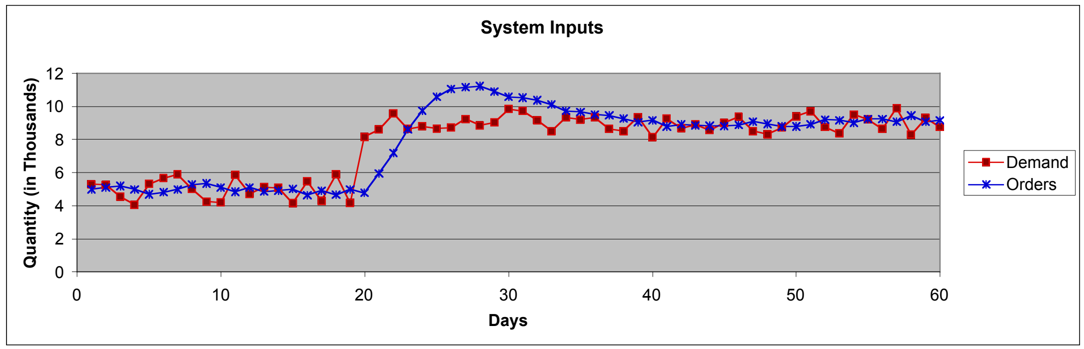
Norm Criteria

RMS Error	7.24
Max Error	18.52

Variability Analysis

Control Error Mean	3.93187885
Control Error Variance	37.65121277
Order Mean	7.89867447
Order Variance	5.222670537
Demand Mean	7.692893686
Demand Variance	3.922814757





Stochastic Gain = 2; +4K/day deterministic disturbance change at k=20; NO setpoint change

Modeling Assignment No. 3

- Add an engineering-based Proportional-Integral-Derivative (PID) decision policy to your previous Excel-based simulation that compares the four EOQ strategies.
- Use your simulation to determine which choice of controlled variable (net stock or inventory position) is “best” suited for this application.
- Evaluate each choice of controlled variable for a 60-day time period; generate both deterministic and stochastic simulations.
- Details provided in the Modeling Assignment No. 3 sheet.

Coming Up

- Tuesday, March 11: Continue working on Modeling Assignment No. 3.

- Thursday, March 13: Modeling Assignment No. 3 due. *Project 1 description will be distributed.*