ECE 100: Introduction to Engineering Design

Modeling Assignment No. 3
Inventory Management Using PID Control

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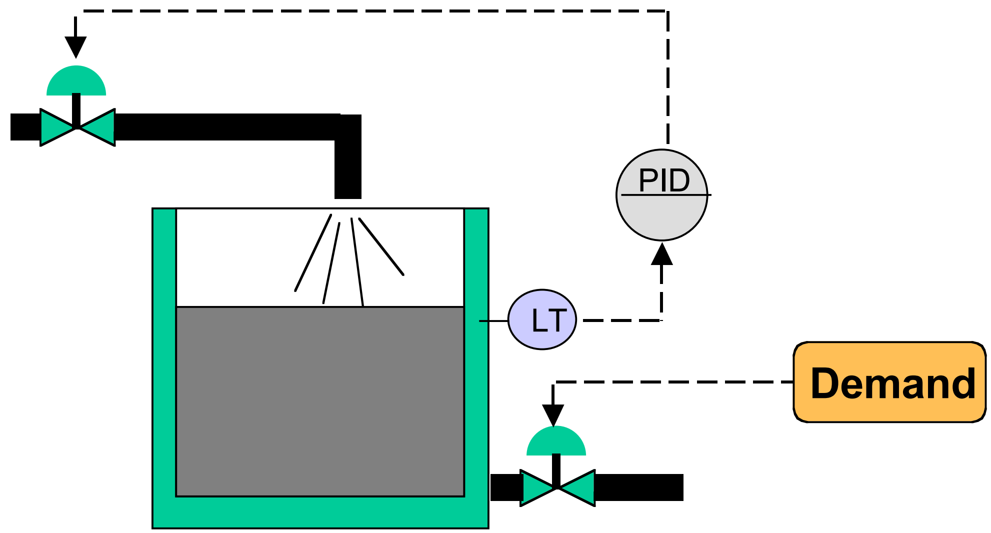
Seating Arrangement

Team 1 / Team 3	Team 4 / Team 5
Team 2 / Team 3	Team 4 / Team 6
Team 7	Team 8 / Team 9
Team 11	Team 8 / Team 10

Some Course Business Items…

- Turn in your Modeling Assignment No. 2 if you have completed these.
- Modeling Assignment No. 2 can be submitted on Thursday, March 6.
- Regarding the myASU problem this weekend: The moral of the story is backup, backup, backup to multiple locations.
- ***Redundancy*** is an important means for achieving reliability in engineering systems
- Our meeting room on Thursday is once again in ECG 224

Feedback-Only Control Strategy



In the feedback-only control problem, order decisions are calculated based only on perceived changes to "level" (e.g., inventory position or equivalent variable).

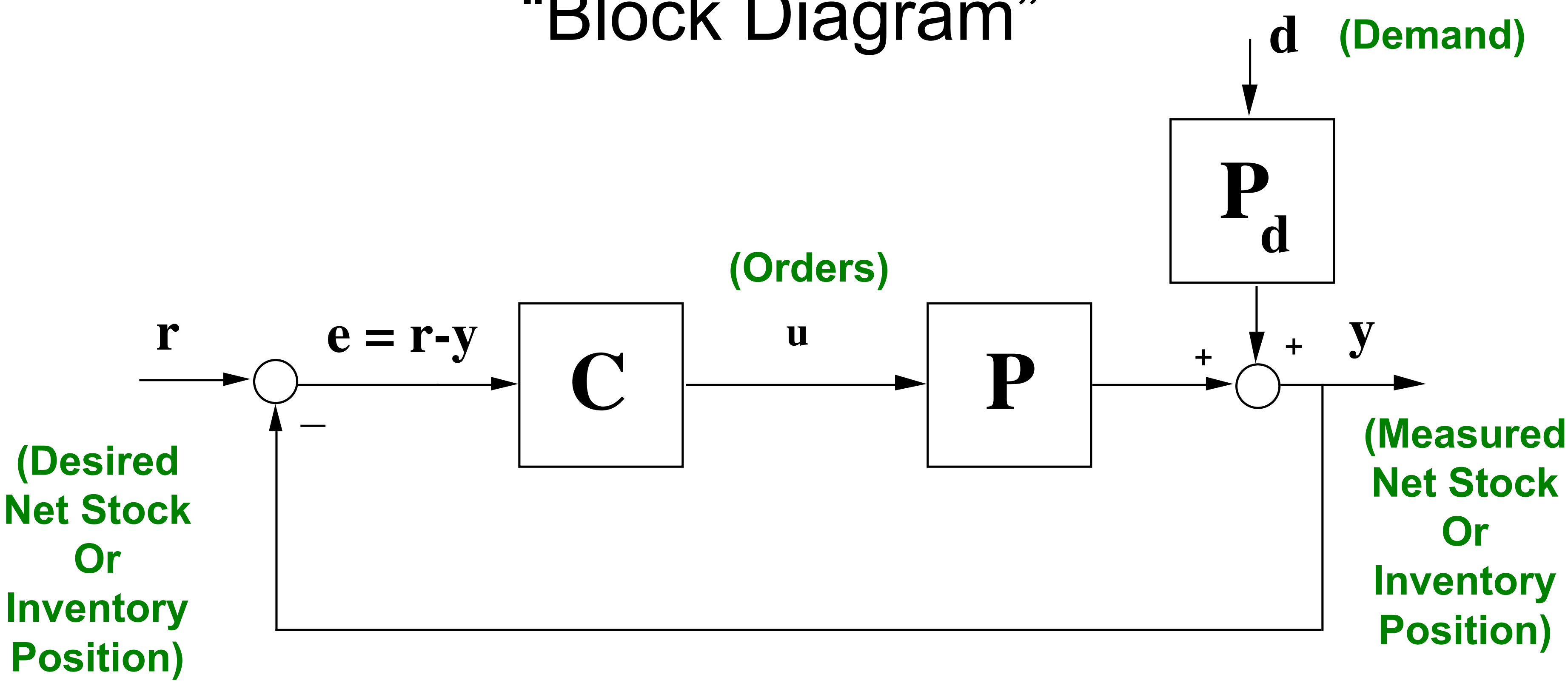
Definitions

- Controlled Variable (y): system variable that we wish to keep at a reference value or *setpoint* (r).
- Control Error (e=r-y): the difference between the controlled variable and the setpoint; we wish to take this to zero.
- Manipulated Variable (u): system variable whose adjustment influences the response of the controlled variable; its value is determined by the controller/decision policy.
- Disturbance Variable (d): system variable that influences the controlled variable response, but cannot be manipulated by the controller; disturbance changes occur external to the system (hence sometimes referred to as *exogeneous* variables)

Supply Chain Management
Inventory Control Problem

- Controlled Variable: For this exercise, evaluate the best choice from the following two candidates:
  - Net Stock *at the beginning of the day* *I(k)*
  - Inventory Position, pre-order, *InvPos(k)*
- Manipulated Variable: Orders *O(k)*
- Disturbance Variable: Demand *D(k)*

Closed-Loop Feedback Control
"Block Diagram"



C = Feedback Controller
P = Process "Transfer Function"
Pd = Disturbance "Transfer Function"

Proportional-Integral-Derivative (PID) with Filter Control

$$u(t) = K_c e(t) + \frac{K_c}{\tau_I} \int_0^t e(t')dt' + K_c \tau_D \frac{de(t)}{dt} - \tau_F \frac{du(t)}{dt}$$

- Choice of Proportional (Kc), Integral (tauI), Derivative (tauD) and Filter (tauF) tuning parameters influence how control error determines the value of the manipulated variable.
- Four "adjustable" tuning *knobs* can represent a tuning challenge for the control designer.
- Please keep in mind that in most applications of control engineering, the choice of tuning parameters is kept away from the end user.

Discrete-Time PID Control

- Our focus will be on a continuous (i.e. daily) review policy
- A difference equation implementation of the IMC-PID with filter controller is shown in the next slide. *This is your engineering-based decision policy!*
- Make sure that all orders are greater than 0 (use MAX command)
- Create a setpoint column in your model, refer to it as *R(k)*
- The controller error e(k) is

  e(k) = R(k) - I(k) (for net stock as controlled var)

  e(k) = R(k) - InvPos(k) (for Inv. Pos. as controlled var)
- Start from steady-state conditions, using the initial net stock or inventory position as your initial setpoint. Set e(0)=e(-1)=e(-2)= 0k/day for k=0

Discrete-Time PID Control (Continued)

$$O(k) = O(k-1) + K_1 e(k) + K_2 e(k-1) + K_3 e(k-2) + K_4 \Delta O(k-1)$$

$$O(k) = O(k-1) + \overbrace{\frac{TK_c}{\tau_F + T}\left(1 + \frac{T}{\tau_I} + \frac{\tau_D}{T}\right)}^{K_1} e(k)$$

$$\overbrace{-\frac{TK_c}{\tau_F + T}\left(1 + \frac{2\tau_D}{T}\right)}^{K_2} e(k-1) + \overbrace{\frac{K_c \tau_D}{\tau_F + T}}^{K_3} e(k-2) + \overbrace{\frac{\tau_F}{\tau_F + T}}^{K_4} \overbrace{(O(k-1) - O(k-2))}^{\Delta O(k-1)}$$

T is the "sampling time" or review period; please keep at 1 (day) for this exercise.

Model-Based PID Tuning

- A model-based tuning rule simplifies the choice of controller tuning parameters.
- We will consider a tuning rule for an "integrating" system which relies on the concept of Internal Model Control (IMC).
- User supplies the order fulfillment time (θ) and only one adjustable parameter (λ), which is inversely proportional to the closed-loop speed of response
  - Increasing λ makes the system sluggish; decreasing λ speeds it up.
  - Be careful about going too fast - it may introduce instability and/or bullwhip…

Model-Based PID Tuning (Continued)

$$\beta = \tau = \frac{\theta}{2}$$

$$K_C = \frac{2(\beta+\lambda)+\tau}{2\beta^2+4\beta\lambda+\lambda^2}$$

$$\tau_I = 2(\beta + \lambda) + \tau$$

$$\tau_D = \frac{2\tau(\beta + \lambda)}{2(\beta + \lambda) + \tau}$$

$$\tau_F = \frac{\beta\lambda^2}{2\beta^2 + 4\beta\lambda + \lambda^2}$$

Operational Objectives of the Inventory Management Control System

- Setpoint Tracking.
- Disturbance Rejection.

Setpoint Tracking

- Refers to the ability of the control system to manipulate orders such that the controlled variable (net stock or inventory position) follows a reference (setpoint) trajectory as closely as possible.

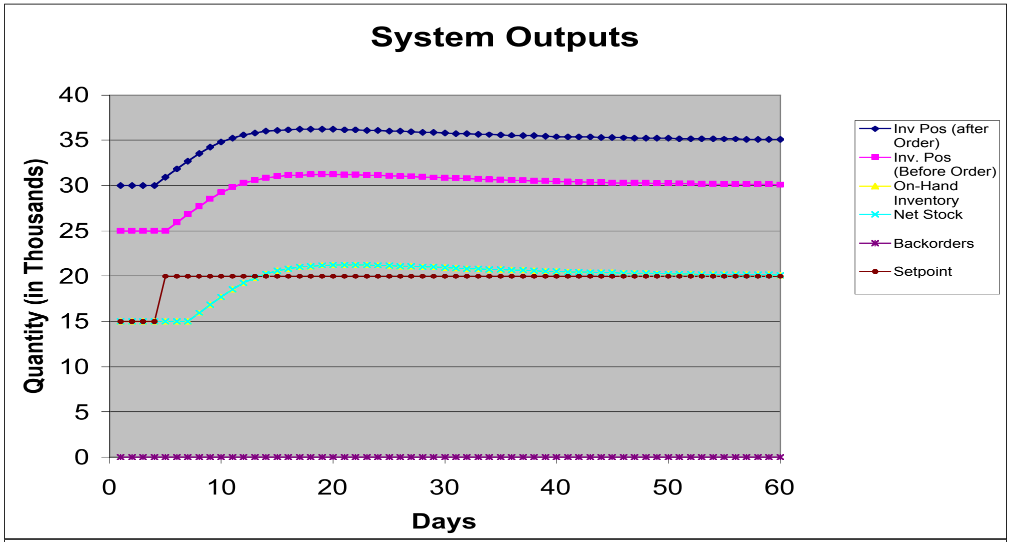
- Changing the inventory position/net stock setpoint from say, 20K units to 10K units while leaving demand unchanged will allow you to observe the setpoint tracking ability of your control system.

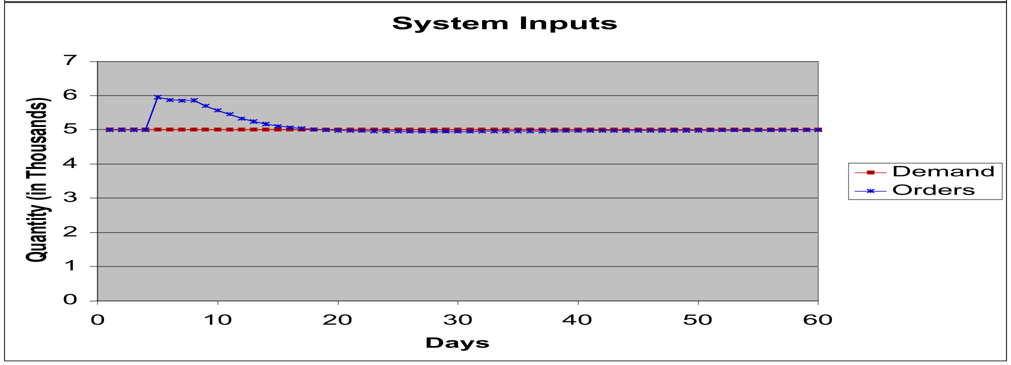
Setpoint Tracking Example

Net Stock as Controlled Variable

(Take Net Stock from 10K units/day to 15K units/day at day = 5)

IMC-PID Decision Policy - Net Stock Controlled Variable	
DE Rivera (Team INSTRUCTOR)	
Initial Net Stock (in K units)	15
Initial Inventory Position (Calculated)	25
Initial (Baseline) Demand (in K units)	5
Initial Orders (O(-2) = O(-1) = O(0))	5
Order Fulfillment Time (Theta)	3
Controller Tuning Parameter (Lambda)	10
Beta	1.5
Tau	1.5
Sampling Time (Ts)	1
Setpoint Change (at k = 5 days)	5
Proportional Gain (Kc)	0.14893617
Integral Time (tauI)	24.5
Derivative Time (tauD)	1.408163265
Filter Time Constant (tauF)	0.911854103
Coeff Kf1	0.190779014
Coeff Kf2	-0.297297297
Coeff Kf3	0.109697933
Coeff Kf4	0.476947536
Inventory Holding Cost ($/K units):	$100
Order Cost ($/order):	$100
Backorder Cost ($/K units):	$1,000
Average On-Hand Inventory (in K units)	19.7
Average Backorders (in K units)	0
Total Orders	60
Total Order Cost	$6,000
Total Inventory Holding Costs	$118,322.30
Total Backorder Costs	$0.00
Total Cost	$124,322.30
Norm Criteria	
RMS Error	1.64
Max Error	5.00





Disturbance Rejection

- Refers to the ability of the control system to manipulate orders such that the controlled variable is kept as close as possible to the setpoint, despite changes in demand.

- For today, please consider using a deterministic demand change:

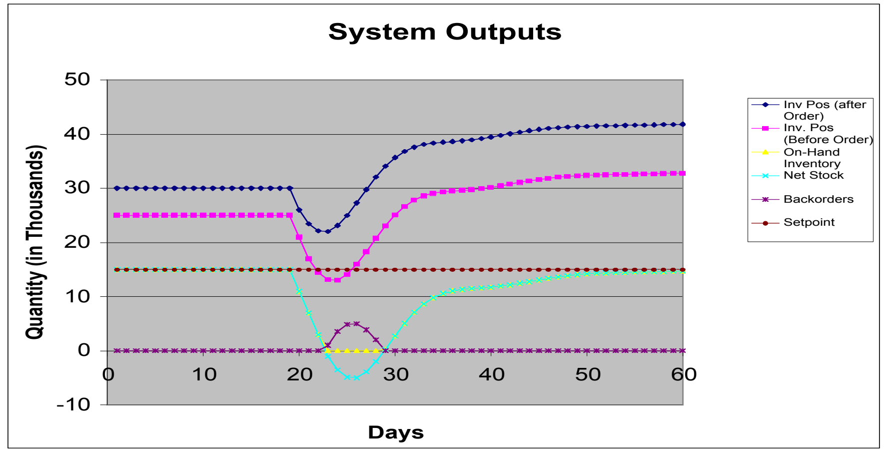
  Total Demand = Nominal (Baseline) Demand + Demand Change

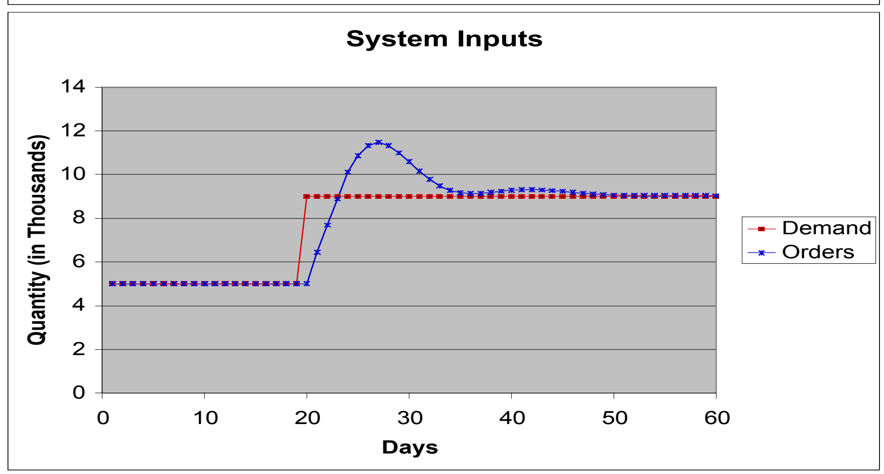
  Next session we will discuss stochastic models and adding a random/stochastic demand change to the problem.

Disturbance Rejection Example
Net Stock as Controlled Variable
(+4k/day demand increase at day 20)

IMC-PID Decision Policy - Net Stock Controlled Variable
DE Rivera (Team INSTRUCTOR)

Initial Net Stock (in K units)	15
Initial Inventory Position (Calculated)	25
Initial (Baseline) Demand (in K units)	5
Initial Orders (O(-2) = O(-1) = O(0))	5
Order Fulfillment Time (Theta)	3
Controller Tuning Parameter (Lambda)	5
Beta	1.5
Tau	1.5
Sampling Time (Ts)	1
Setpoint Change (at k = 5 days)	0
Proportional Gain (Kc)	0.243697479
Integral Time (tauI)	14.5
Derivative Time (tauD)	1.344827586
Filter Time Constant (tauF)	0.630252101
Coeff Kf1	0.360824742
Coeff Kf2	-0.551546392
Coeff Kf3	0.201030928
Coeff Kf4	0.386597938
Inventory Holding Cost ($/K units):	$100
Order Cost ($/order):	$100
Backorder Cost ($/K units):	$1,000
Average On-Hand Inventory (in K units)	11.4
Average Backorders (in K units)	0.338760121
Total Orders	60
Total Order Cost	$6,000
Total Inventory Holding Costs	$68,428.28
Total Backorder Costs	$20,325.61
Total Cost	$94,753.89
<u>**Norm Criteria**</u>	
RMS Error	7.05
Max Error	19.99





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Assessing Closed-Loop Performance

- Deterministic Measures
  - Bounded Input, Bounded Output (BIBO) stability. Bounded changes in demand result in bounded changes in orders and inventories. In a stable response, there is c*onvergence* to a steady-state, as opposed to *divergence*.
  - Response characteristics.
    - Shape of response (i.e., smooth or oscillatory)
    - Offset (control error does not go to zero after long enough time)
    - Settling time
    - Overshoot/undershoot

*Please examine the **clperformance.pdf** handout (posted on the course website) for more details*

Assessing Closed-Loop Performance (Continued)

- Deterministic Measures (*Norm* Criteria)

The Root-Mean-Square (RMS) control error is computed as

$$RMSerr = \left(\frac{1}{N}\sum_{k=1}^{N} e^2(k)\right)^{1/2} = \left(\frac{1}{N}\sum_{k=1}^{N}(r(k)-y(k))^2\right)^{1/2}$$

while the maximum (MAX) control error consists of the largest absolute magnitude error

$$MAXerr = \max_k |e(k)| \qquad k = 1, \cdots, N$$

N is the total number of days in the simulation run.

Use SQRT, SUM, MAX, and ABS commands to implement these measures in your spreadsheet! You will need to create new columns to compute these properly.

Modeling Assignment No. 3

- Add an engineering-based Proportional-Integral-Derivative (PID) decision policy to your previous Excel-based simulation that compares the four EOQ strategies. *Create new worksheets for this purpose.*
- One worksheet should evaluate using net stock as CV, the other should use inventory position.
- Use the response characteristics and RMS/MAX error criteria to determine which choice of controlled variable (net stock or inventory position) is "best" suited for this application.
- Evaluate each policy for a 60-day time period.
- A written description will be provided on Thursday, which will include a description of the stochastic component of the problem.

In-Class

- Complete Modeling Assignment No. 2 (if you have not done so already) and begin working on Modeling Assignment No. 3
- Discuss as a team what additional features are required for the PID-based decision policy. Break up the work appropriately.
- Your ticket out is to reproduce the setpoint tracking and disturbance rejection responses shown in this presentation.

Coming Up

- Thursday, March 6: Continued work on Modeling Assignment No. 3. We will *meet again in ECG 224!*

- Tuesday, March 11: Complete modeling assignment No. 3. Prepare for Project No. 1.

- Thursday, March 13: Project No. 1 description distributed.