

A Dynamical Systems Model for Understanding Behavioral Interventions for Weight Loss

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Abstract. We propose a dynamical systems model that captures the daily fluctuations of human weight change, incorporating both physiological and psychological factors. The model consists of an energy balance integrated with a mechanistic behavioral model inspired by the Theory of Planned Behavior (TPB); the latter describes how important variables in a behavioral intervention can influence healthy eating habits and increased physical activity over time. The model can be used to inform behavioral scientists in the design of optimized interventions for weight loss and body composition change.

Keywords: behavioral interventions, theory of planned behavior, dynamical systems, energy balance, weight loss.

1 Introduction

Obesity rates in the United States have increased substantially in the past few decades [13]. Because obesity represents a preventable cause of premature morbidity and death, much research activity has been devoted to understanding its causes, and a number of diverse solutions have been proposed [6,9,19]. Some of these solutions have major disadvantages, for instance, surgery and extreme diets usually lead to a regain of the weight lost within 1 to 5 years [2,11]. Solutions leading to permanent weight loss require sustained lifestyle changes in an individual; consequently, developing optimized behavioral interventions that promote healthy eating habits and increased physical activity represents a problem of both fundamental and practical importance.

The primary goal of this paper is to improve the understanding of behavioral weight change interventions by expressing these as dynamical systems. Dynamical systems modeling has been used in the analysis of novel behavioral interventions such as adaptive interventions [17]. Dynamical systems modeling considers how important system variables (e.g., proximal and distal outcomes, mediators)

respond to changes in input variables (e.g., intervention dosages, external characteristics) over time. A dynamical systems model can be used to answer questions regarding what variables to measure, how often, and the speed and shape of the outcome responses as a result of decisions regarding the timing, spacing, and dosage levels of intervention components.

To achieve this goal, we develop in this paper a dynamical systems model for daily weight change incorporating both physiological and psychological considerations. For the physiological component, we rely on the concept of energy balance to obtain a model that describes the net effect of energy intake from food minus energy consumption, the latter which includes physical activity. For the psychological component, we present a model for the dynamics of diet and exercise behavior. This model explains how intentions, social norms, attitudes, and other system variables that are impacted by an intervention result in healthy eating habits and increased physical activity over time. A model based on the Theory of Planned Behavior (TPB) is used for this purpose. The dynamical systems model for weight change can be used to answer questions regarding how much to eat, what kinds of food to eat, how much physical activity to undertake, and how long it will take before desired weight loss goals are achieved.

The paper is organized as follows: Section 2 presents the energy balance model, while Section 3 gives a brief description of the Theory of Planned Behavior (TPB) and presents a mechanistic dynamical model for TPB based on fluid analogies. Section 4 describes a representative simulation from this model and discusses the role and importance of some of the parameters in the model. Finally, Section 5 summarizes our main conclusions and discusses areas of current and further study.

2 Energy Balance Model

In this section, we present the energy balance model used for our investigation. This model is based on a three-compartment model proposed in [10,11,12]. The normal daily energy balance $EB(t)$ is described as follows:

$$EB(t) = EI(t) - EE(t), \quad (1)$$

where $EI(t)$ is the energy intake and $EE(t)$ is the energy expenditure at time t , measured at daily intervals in this study. The energy intake EI , expressed in kilocalories (kcal), is modeled using the Atwater methods of energy calculation resulting from carbohydrate intake (CI), fat intake (FI), and protein intake (PI), all expressed in grams/day [14]:

$$EI(t) = a_1 CI(t) + a_2 FI(t) + a_3 PI(t) \quad (2)$$

Here $a_1 = 4$ kcal/gram, $a_2 = 9$ kcal/gram, and $a_3 = 4$ kcal/gram. The energy expenditure EE , expressed in kcal, is calculated as follows:

$$EE(t) = \beta EI(t) + \delta BM + K + \gamma_{LM} LM(t) + \gamma_{FM} FM(t) + \eta_{FM} \frac{dFM}{dt} + \eta_{LM} \frac{dLM}{dt} \quad (3)$$

The first term, $\beta EI(t)$, denotes the energy expended in processing food ($\beta = 0.24$), δ is the physical activity coefficient (expressed in kcal/kg), $\gamma_{LM} = 22$ kcal/kg/d, $\gamma_{FM} = 3.2$ kcal/kg/d, $\eta_{LM} = 230$ kcal/kg and $\eta_{FM} = 180$ kcal/kg are all coefficients for the calculation of the Resting Metabolic Rate (RMR) which depends on the lean mass LM and the fat mass FM . The constant K accounts for initial energy balance conditions and is determined by solving equation (3) assuming an initial steady-state at $t = 0$; the steady-state is denoted by a bar over any time-dependent variable ($\overline{EI} - \overline{EE} = 0$, with $d\overline{FM}/dt = d\overline{LM}/dt = 0$ by definition of steady-state):

$$K = -\gamma_{LM}\overline{LM} - \gamma_{FM}\overline{FM} - \delta\overline{BM} + \overline{EI}(1 - \beta). \quad (4)$$

The three-compartment model for fat mass FM , lean mass LM and extra-cellular fluid volume ECF is summarized as follows:

$$\frac{dFM(t)}{dt} = \frac{(1 - p(t))EB(t)}{\rho_{FM}} \quad (5)$$

$$\frac{dLM(t)}{dt} = \frac{p(t)EB(t)}{\rho_{LM}} \quad (6)$$

$$\frac{dECF}{dt} = \frac{\Delta Na_{diet} - \xi_{Na}(ECF - ECF_{init}) - \xi_{CI}(1 - CI/CI_b)}{[Na] \tau_{Na}}, \quad (7)$$

where $\rho_{FM} = 9400$ kcal/kg, $\rho_{LM} = 1800$ kcal/kg and p is given by the Forbes formula [8]:

$$p = \frac{C}{(C + FM)}; \quad C = 10.4 \frac{\rho_{LM}}{\rho_{FM}}. \quad (8)$$

For the extracellular fluid volume (in ml), ΔNa_{diet} is the change on sodium in mg/d, CI_b is the baseline carbohydrate intake, $[Na] = 3.22$ mg/ml, $\xi_{Na} = 3$ mg/ml/d, $\xi_{CI} = 4000$ mg/d, ECF_{init} is the initial ECF volume and $\tau_{Na} = 2$, which corresponds to a time constant of two days. Finally, the body mass is given by the sum of fat mass FM , lean mass LM and extracellular fluid volume ECF :

$$BM(t) = FM(t) + LM(t) + ECF(t). \quad (9)$$

In summary, the mechanistic energy balance leads to a dynamical model with EI (composed of CI , FI , and PI), δ , and ΔNa_{diet} as inputs, and FM , LM , and ECF as outputs; the outputs add up to total body mass (BM).

3 Behavioral Model

The Theory of Planned Behavior (TPB) [1] is an accepted and broadly used paradigm for describing the relationship between behaviors and intentions, attitudes, norms, and perceived control in behavioral science. Many studies have relied on TPB and its forerunner the Theory of Reasoned Action (TRA) [7] to describe behavioral changes for healthy eating [3] and exercising [5]. Behavior is the observable response in a given situation with respect to a given target, while intention is an indication of the readiness of a person to perform a given behavior. According to TPB, intention is influenced by the following components:

Attitude Toward the Behavior: This is the degree to which performing the behavior is positively or negatively valued. It is determined by the *strength of beliefs about the outcome* and the *evaluation of the outcome*.

Subjective Norm: This is the perceived social pressure to engage or not engage in a behavior. It is determined by the strength of the beliefs what people want the person to do, also called *normative beliefs*, and the desire to please people, also called *motivation to comply*.

Perceived Behavioral Control: This reflects the perception of the ability to perform a given behavior, i.e. the beliefs about the presence of factors that may facilitate or impede performance of the behavior. It is determined by the *strength of each control belief* and the *perceived power of the control factor*.

A standard mathematical representation for TPB relies on Structural Equation Modeling (SEM) [4]. The field of SEM is substantial, but in this work we limit ourselves to a special case of SEM called path analysis. The main characteristics of path analysis models is that they do not contain latent variables, i.e., all problem variables are observed, and the independent variables are assumed to have no measurement error [16]. The TPB represented as a path analysis model with a vector η of endogenous variables and a vector ξ of exogenous variables is expressed as follows:

$$\eta = \mathbf{B} \eta + \mathbf{\Gamma} \xi + \zeta \quad (10)$$

$$\begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \beta_{41} & \beta_{42} & \beta_{43} & 0 & 0 \\ 0 & 0 & \beta_{53} & \beta_{54} & 0 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \end{bmatrix} + \begin{bmatrix} \gamma_{11} & 0 & 0 \\ 0 & \gamma_{22} & 0 \\ 0 & 0 & \gamma_{33} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} + \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \end{bmatrix} \quad (11)$$

where \mathbf{B} and $\mathbf{\Gamma}$ are matrices of β_{ij} and γ_{ij} regression weights, respectively, and ζ is a vector of disturbance variables. Figure 1 shows the intention-behavior TPB path analysis model for equation (11). Typically, the principles of TPB assume that the attitude toward the behavior ξ_1 , the subjective norms ξ_2 and the perceived behavioral control ξ_3 are estimated using the expectancy-value model, which considers the sum over the person's behavioral beliefs, normative beliefs and control beliefs, respectively, that are accessible at the time. However, for simplicity of presentation and without loss of generality, we consider only one exogenous variable per compartment in this paper. Thus,

$$\xi_1 = b_1 \times e_1 \quad (12)$$

$$\xi_2 = n_1 \times m_1 \quad (13)$$

$$\xi_3 = c_1 \times p_1, \quad (14)$$

where b_1 is the behavioral belief, e_1 the evaluation of the outcome, n_1 the normative belief, m_1 the motivation to comply, c_1 the control belief and p_1 the power of the control belief.

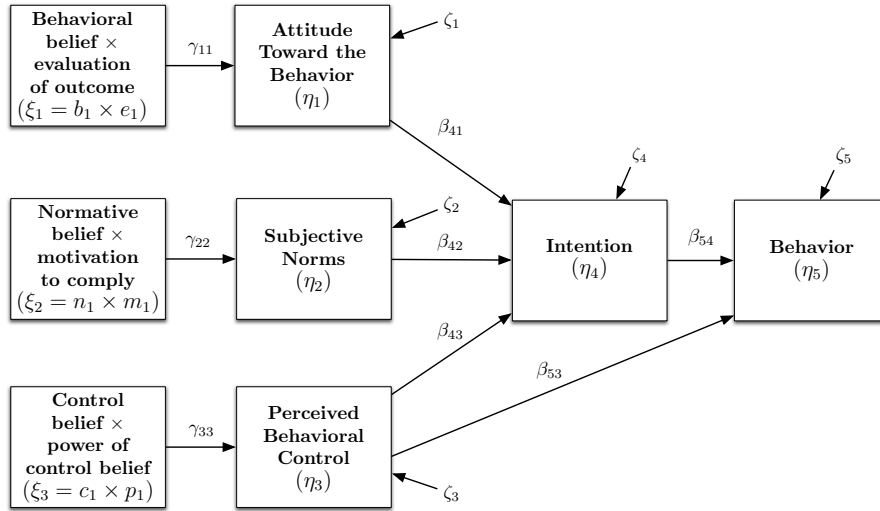


Fig. 1. Path analysis diagram for the Theory of Planned Behavior (TPB) with three exogenous variables ξ_i , five endogenous variables η_i , regression weights γ_{ij} and β_{ij} and disturbances ζ_i

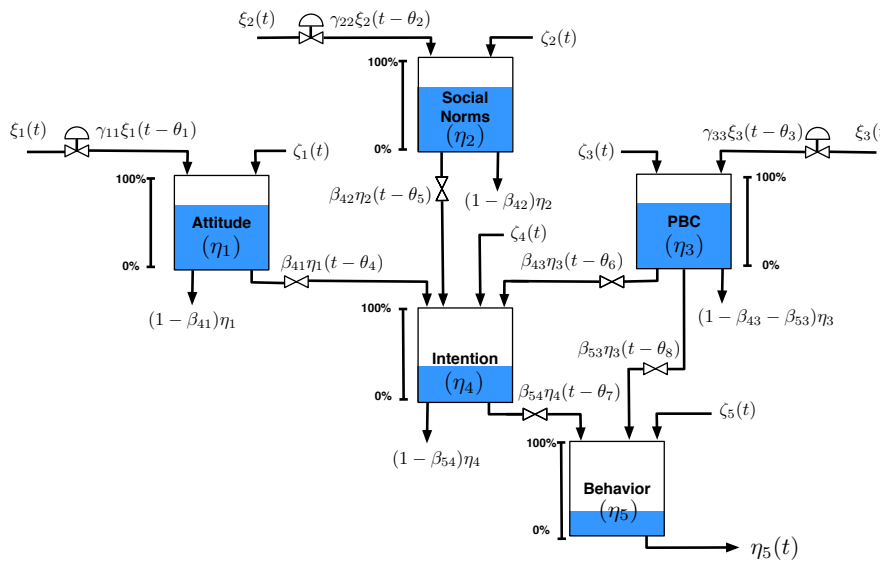


Fig. 2. TPB as a dynamical system, representation based on a fluid inventory control system. Time delays are modeled by $\theta_1, \dots, \theta_8$. Additional parameters as in Figure 1.

3.1 Dynamic Fluid Analogy for TPB

The classical TPB model as expressed in equation (10) represents a static (i.e., steady-state) system that does not capture any changing behavior over time. In order to expand the TPB model to include dynamic effects, we propose the use of a fluid analogy which parallels the problem of inventory management in supply chains [18]. This analogy is expressed diagrammatically in Figure 2. We consider a dynamic fluid analogy of TPB with five *inventories*: attitude η_1 , social norms η_2 , perceived behavioral control η_3 , intention η_4 and behavior η_5 . Each inventory is replenished by inflow streams and depleted by outflow streams. The path diagram model coefficients $\gamma_{11}, \dots, \gamma_{33}$ are the inflow resistances and $\beta_{41}, \dots, \beta_{54}$ are the outflow resistances, which can be physically interpreted as those fractions of the inventories of the system that serve as inflows to the subsequent layer in the path analysis model.

To generate the dynamical system description we apply the principle of conservation of mass to each inventory, where accumulation corresponds to the net difference between mass inflows and outflows:

$$\text{Accumulation} = \text{Inflow} - \text{Outflow} \quad (15)$$

Relying on the rate form for equation (15) leads to a system of differential equations according to:

$$\tau_1 \frac{d\eta_1}{dt} = \gamma_{11}\xi_1(t - \theta_1) - \eta_1(t) + \zeta_1(t) \quad (16)$$

$$\tau_2 \frac{d\eta_2}{dt} = \gamma_{22}\xi_2(t - \theta_2) - \eta_2(t) + \zeta_2(t) \quad (17)$$

$$\tau_3 \frac{d\eta_3}{dt} = \gamma_{33}\xi_3(t - \theta_3) - \eta_3(t) + \zeta_3(t) \quad (18)$$

$$\tau_4 \frac{d\eta_4}{dt} = \beta_{41}\eta_1(t - \theta_4) + \beta_{42}\eta_2(t - \theta_5) + \beta_{43}\eta_3(t - \theta_6) - \eta_4(t) + \zeta_4(t) \quad (19)$$

$$\tau_5 \frac{d\eta_5}{dt} = \beta_{54}\eta_4(t - \theta_7) + \beta_{53}\eta_3(t - \theta_8) - \eta_5(t) + \zeta_5(t), \quad (20)$$

where, following equations 12-14, $\xi_1(t) = b_1(t)e_1(t)$, $\xi_2(t) = n_1(t)m_1(t)$, $\xi_3(t) = c_1(t)p_1(t)$, and ζ_1, \dots, ζ_5 are zero-mean stochastic signals. The dynamical system representation according to equations 16 through 20 includes all the path analysis model parameters and is enhanced by the presence of *time delays* $\theta_1, \dots, \theta_8$ (which model the lag in the inflow/outflow process) and *time constants* τ_1, \dots, τ_5 (which model the capacities of the inventories) for each inventory in the system. These parameters can be used to determine the speed at which an individual or population can transition between values for η_1, \dots, η_5 as a result of changes in the variables ξ_i . A number of important points of interest are summarized below:

1. At steady-state, i.e., when $\frac{d\eta_i}{dt} = 0$, equations 16 through 20 reduce to the path analysis model in equation (11) *without approximation*.

2. The path analysis model coefficients γ_{ij} and β_{ij} correspond directly to *gains* in the dynamical system.
3. The outflow resistances from the inventory PBC are subject to the constraint: $\beta_{53} + \beta_{54} \leq 1$.
4. The dynamical model representation is not limited to describing single subjects. Typically, path analysis models are naturally estimated cross-sectionally from data obtained from multiple participants. Dynamical system model parameters can similarly be estimated over a group or cohort, but doing so will require availability of repeated measurements of system variables over time.

4 Simulation Study

The overall dynamical systems model for the behavioral intervention integrates the model described in Section 2 and the TPB dynamical model described in Section 3 by having the outputs of the TPB model serve as inputs to the mechanistic energy balance model. This enables the impact of the intervention to be observed in both psychological and physical outcome variables over time.

The simulation study consists of examining the effects over time of an intervention promoting healthy eating habits and increased physical activity for a representative male participant at the following initial conditions: $BM = 100$ kg, $FM = 30$ kg, $LM = 45$ kg and $ECF = 25$ liters. Figure 3 (top) shows the responses of the intervention on TPB models for energy intake behavior (EI-TPB) and physical activity behavior (PA-TPB). Figure 3 (bottom) shows the changes in the body compartments corresponding to these interventions. We consider a scenario that as a result of the intervention the intensity of beliefs about healthy eating habits increases from $b_1 = 7$ to $b_1 = 10$. This change leads to an increase on the exogenous variable ξ_1 in the EI-TPB system. In the same manner, we assume that as a result of the intervention there is a change in the beliefs about proper exercising from $b_1 = 1$ to $b_1 = 3$, which also leads to an increase on the variable ξ_1 but in the PA-TPB system. No outflow from the inventory PBC to the inventory behavior is considered for both behavioral models, i.e. $\beta_{53} = 0$. For this simulation study we consider the following three sub-scenarios:

- i) The participant completely assimilates the intervention and immediately starts changing eating habits and exercising. This means a rapid time constant in the attitude inventory ($\tau_1 = 0.1$), no depletion in the intention inventory ($\beta_{41} = 1$) and no delay in the behavior inventory ($\theta_7 = 0$).
- ii) The participant partially and slowly assimilates the intervention. This means a slow time constant in the attitude ($\tau_1 = 20$ days), depletion on intention of $\beta_{41} = 0.5$ (only 50% of the outflow makes the next inventory) and a delay in behavior of $\theta_7 = 15$ days.
- iii) The same scenario as for case ii) but with disturbances on the attitude towards healthy eating and exercising. These disturbances are represented as white noise signals on energy intake EI and physical activity δ with means and variances $N(0, 20)$ and $N(0, 50)$, respectively.

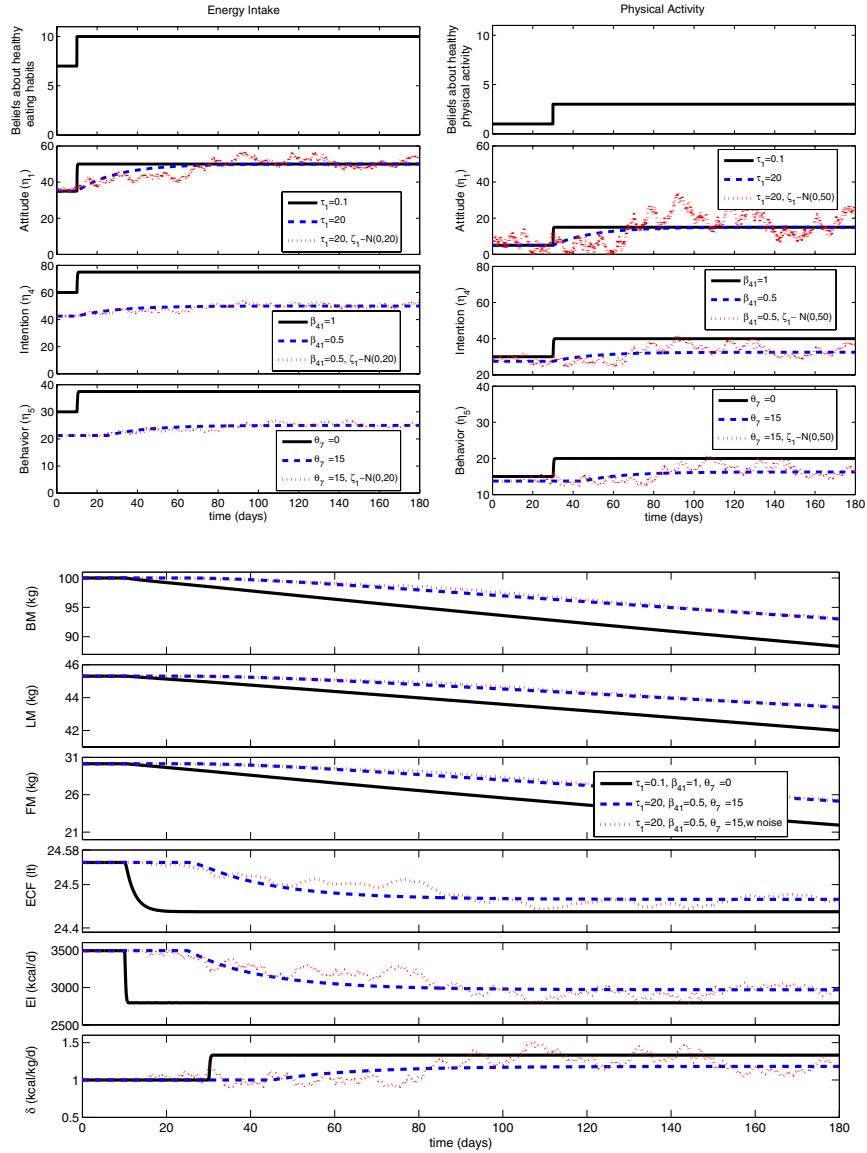


Fig. 3. (top) Step responses for the inventory systems of energy intake behavior EI-TPB and physical activity behavior PA-TPB, for interventions on beliefs about the outcome b_1 ; (bottom) changes on body compartments and total effect of intervention on EI and PA. Simulations for the following intervention cases: (i) complete assimilation $\tau_1 = 0.1, \beta_{41} = 1, \theta_7 = 0$, (ii) partial assimilation $\tau_1 = 20, \beta_{41} = 0.5, \theta_7 = 15$, and (iii) partial assimilation as in case (ii) but with noise $\zeta_1 \sim N(0, 20)$ (for EI), and $\zeta_1 \sim N(0, 50)$ (for PA). Further parameters: $\xi_2 = \xi_3 = 50, \theta_1 = \dots = \theta_6 = 0, \theta_8 = 0, \tau_2 = \dots = \tau_3 = 0.1, \gamma_{ij} = 1, \beta_{42} = \beta_{43} = \beta_{54} = 0.5$ and $\beta_{53} = 0$.

After a step change in the variable ξ_1 is introduced, the magnitude level of the inventories for attitude, intention and behavior change, respectively. Observe that the attitude response in scenario (ii) takes a larger number of time steps to reach the steady-state when compared to scenario (i). This occurs because of the different τ_1 values, the larger value of τ_i represents a longer transition of the system to the new steady state. Moreover, the level of intention in scenario (ii) is much lower than in scenario (i). The smaller β_{ij} the larger the depletion. Finally, the change in behavior in scenario (ii) starts much after the change in behavior in scenario (i). The larger the value for θ_i , the longer the delay.

A number of conclusions can be drawn from these simulation results. The most significant is the contrast between the results of case (i) versus case (ii) after a six-month time period. The weight for the participant in case (i) decreases by almost 10 kg over the six month period, whereas for case (ii) the weight loss is only 7 kg. The behavioral “lag” has resulted in a substantially lower achievable weight loss for the participant. However, despite the presence of stochastic disturbances in case (iii), these do not result in significant differences on total body weight loss when compared to case (ii).

5 Summary and Conclusions

A dynamical systems model for a behavioral intervention associated with weight loss has been proposed, which provides a potentially useful framework for understanding and optimizing this class of interventions. By being able to test the effect of intervention components on outcomes of interest over time, an intervention scientist can use this information to optimally decide on the ordering and strength of intervention components, and better predict both the inter- and intra-individual variability that will be reflected in these interventions.

An extended version of the dynamic TPB model has been developed where all endogenous variables are latent as opposed to observed variables; it has not been presented in this paper for reasons of brevity.

The simulation results point to the need for data from experimental trials or observational studies that can be used to estimate parameter coefficients in these models and validate the modeling framework. We are currently exploring how methods from the field of system identification [15], coupled with data resulting from participant diaries or ecological momentary assessment, can be used for this purpose. A long-term goal is to develop adaptive behavioral interventions for preventing weight gain or loss in patients with obesity or malnutrition, relying on control systems engineering principles [17].

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