











parental function to 65% and a simultaneous step unmeasured disturbance  $D(k) = 4$ . In both the cases, the MPC tuning parameters  $Q_y = 1$ ,  $Q_{\Delta u} = 0.1$ ,  $Q_u = Q_d = Q_z = 0$ ,  $(\alpha_r, f_a) = (0, 0.3)$ , the prediction horizon  $p = 40$  and control horizon  $m = 10$  are used. The *Tomlab-CPLEX* solver is used to solve resulting *miqp* optimization problems. From the figure it can be seen that the controller designed using the MoD approach is able to quickly achieve the desired setpoint, and stabilizes the system at the setpoint. In contrast, the controller relying on the linear ARX model oscillates around setpoint. In addition, the proposed algorithm produces less variation in the manipulated variable and provides uniform performance. This fact is also confirmed by the performance matrices  $J_e$  and  $J_{\Delta}$  given below:

$$J_e = \sum_{k=1}^{t/T_s} (PF(k) - PF^{goal})^T (PF(k) - PF^{goal}) \quad (34)$$

$$J_{\Delta} = \sum_{k=1}^{t/T_s} (I(k) - I(k-1))^T (I(k) - I(k-1)) \quad (35)$$

where  $t$  represents total simulation time and  $T_s$  is a sampling time. The performance matrices  $J_e$  and  $J_{\Delta}$  represent measure of cumulative deviation of parental function from the goal and measure of cumulative variation in the intervention dosages, respectively. For the MoD approach, values of  $J_e$  and  $J_{\Delta}$  are  $7.84 \times 10^3$  and  $1.11 \times 10^4$ , respectively, while using the linear ARX model based MPC, these values are  $9.05 \times 10^3$  and  $1.55 \times 10^4$ , respectively. Thus, it can be concluded that the proposed algorithm yields superior performance and is suitable for the control of nonlinear hybrid systems.

## V. SUMMARY

Applications of hybrid systems are becoming increasingly common in many fields. Recently, control engineering principles have been proposed for adaptive behavioral interventions [21]; these systems are naturally hybrid in nature. In this work, a Model-on-Demand Predictive Control (MoDPC) approach for control of nonlinear hybrid systems and its application to a simulated adaptive behavioral intervention are presented. The formulation uses a Model-on-Demand approach to obtain a local MLD model for the nonlinear hybrid system at each time step. MoD is a data-centric approach that uses a small neighborhood data around current operating point characterized by the regressor vector. The local MLD model generated by MoD estimator is then used to specify a model predictive control law that relies on multiple-degree-of-freedom tuning parameters [4]. Multiple-degree-of-freedom tuning enables the speed of disturbance rejection and setpoint tracking affecting each output to be adjusted individually; this has intuitive appeal. The applicability and efficiency of proposed formulation is demonstrated on a hypothetical intervention problem intended for improving parental function in at-risk children. This problem exhibits nonlinear dynamics with inherent discrete events. From the simulation results, it can be concluded that the proposed MoDPC is useful for the control of nonlinear hybrid systems,

displaying acceptable performance levels while simplifying the task of modeling.

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## REFERENCES

- [1] A. Bemporad and M. Morari. Control of systems integrating logic, dynamics, and constraints. *Automatica*, 35(3):407 – 427, 1999.
- [2] N. N. Nandola and S. Bhartiya. A multiple model approach for predictive control of nonlinear hybrid systems. *J. Process Control*, 18(2):131 – 148, 2008.
- [3] N. N. Nandola and S. Bhartiya. A computationally efficient scheme for model predictive control of nonlinear hybrid systems using generalized outer approximation. *Ind. Eng. Chem. Res.*, 48(12):5767 – 5778, 2009.
- [4] N. N. Nandola and D. E. Rivera. A novel model predictive control formulation for hybrid systems with application to adaptive behavioral interventions. In *ACC*, volume FrC01.5, pages 6286 – 6292, Baltimore, Maryland, USA, June 30-July 02 2010.
- [5] L. M. Collins, S. A. Murphy, and K. L. Bierman. A conceptual framework for adaptive preventive interventions. *Prevention Science*, 5(3):185–196, 2004.
- [6] L. Ljung. *System Identification Theory For the User*. PTR Prentice Hall, New Jersey, second edition, 1999.
- [7] R. Vidal. Recursive identification of switched ARX systems. *Automatica*, 44(9):2274 – 2287, 2008.
- [8] P. Egbunonu and M. Guay. Identification of switched linear systems using subspace and integer programming techniques. *Nonlinear Anal. Hybrid Syst*, 1(4):577 – 592, 2007.
- [9] J. Roll, A. Bemporad, and L. Ljung. Identification of piecewise affine systems via mixed-integer programming. *Automatica*, 40(1):37 – 50, 2004.
- [10] G. Ferrari-Trecate, M. Muselli, D. Liberati, and M. Morari. A clustering technique for the identification of piecewise affine systems. *Automatica*, 39(2):205 – 217, 2003.
- [11] N. N. Nandola and S. Bhartiya. Hybrid system identification using a structural approach and its model based control: An experimental validation. *Nonlinear Anal. Hybrid Syst*, 3(2):87 – 100, 2009.
- [12] G. Cybenko. Just-in-time learning and estimation. In S. Bittani and G. Picci, editors, *Identification, Adaptation, Learning*, NATO ASI, pages 423 – 434. Springer, 1996.
- [13] M. Braun. *Model-on-demand nonlinear estimation and model predictive control: Novel methodologies for process control and supply chain management*. PhD thesis, Arizona State University, USA, 2001.
- [14] A. Stenman. *Model on demand: Algorithms, analysis and applications*. PhD thesis, Linköping University, Sweden, 1999.
- [15] J. Roll, A. Nazin, and L. Ljung. Nonlinear system identification via direct weight optimization. *Automatica*, 41(3):475 – 490, 2005. Data-Based Modelling and System Identification.
- [16] M. Braun, D. E. Rivera, and A. Stenman. A model-on-demand identification methodology for nonlinear process systems. *Int. J. Control*, 74(18):1708 – 1717, 2001.
- [17] M. Braun, A. Stenman, and D. E. Rivera. Model-on-demand model predictive control toolbox. <http://cseel.asu.edu/MoDMPCToolbox>, 2002.
- [18] J. H. Lee and Z. H. Yu. Tuning of model predictive controllers for robust performance. *Comput. Chem. Eng.*, 18(1):15 – 37, 1994.
- [19] M. Morari and E. Zafriou. *Robust Process Control*. Englewood Cliffs, NJ: Prentice-Hall, 1989.
- [20] Conduct Problems Prevention Research Group. A developmental and clinical model for the prevention of conduct disorders: The Fast Track program. *Development and Psychopathology*, 4:509–528, 1992.
- [21] D. E. Rivera, M. D. Pew, and L. M. Collins. Using engineering control principles to inform the design of adaptive interventions: a conceptual introduction. *Drug and Alcohol Dependence*, 88(2):S31–S40, 2007.