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Hybrid Model Predictive Control Applied to Production-Inventory Systems ⋆

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Abstract: Hybrid production-inventory systems are characterized by discrete decisions on production levels and/or capacity. These systems have broad applicability to important, emerging applications of process control concepts, among them time-varying adaptive behavioral interventions and supply chain management. This paper examines the usefulness of hybrid model predictive control (HMPC) in these two novel application settings. In a hypothetical adaptive behavioral intervention inspired by *Fast Track* (a preventive intervention for reducing conduct disorder in at-risk children), HMPC is presented as a means to improve the assignment of frequency of home-based counseling visits to families with low parental function. In supply chain management, the usefulness of HMPC for assigning production capacity in an inventory control problem under conditions of varying customer demand is presented. These problems are modeled as mixed logical dynamical (MLD) systems, with HMPC consisting of a Mixed Integer Quadratic Program (MIQP) that employs a three-degree-of-freedom parametrization for achieving ease of tuning and facilitating robust performance under uncertainty.

Keywords: Hybrid systems; Model Predictive Control; production-inventory systems; adaptive behavioral interventions; supply chain management

1. INTRODUCTION

Hybrid systems are characterized by interactions between continuous and discrete dynamics. The term hybrid has also been applied to describe processes that involve continuous dynamics and discrete (logical) decisions (Bemporad and Morari, 1999; Nandola and Bhartiya, 2008). Hybrid systems occur in many diverse settings; these include manufacturing, automotive systems, and process control. In recent years, significant emphasis has been given to modeling, identification, control and optimization of linear and nonlinear hybrid systems (Nandola, 2009). The review paper by Camacho et al. (2010) notes that despite the considerable interest within the control engineering community for model predictive control for hybrid systems, the field has not been fully developed, and many open challenges remain. One of these is the application to new areas outside of the industrial community, and the the need for novel formulations that can be effectively used in noisy, uncertain environments. This paper demonstrates the application of hybrid MPC to two non-traditional areas that can be conceptualized as production-inventory systems: adaptive interventions in behavioral health, and inventory management in supply chains. These problems are hybrid in nature and display performance requirements that demand a flexible control formulation. We consider a novel Three-Degree-of-Freedom (3-DoF) Model Predictive Control formulation developed by Nandola and Rivera (2010), which offers performance and ease of tuning that is amenable for robustification in hybrid systems.

Production-inventory control is a classical problem in enterprise systems that has application in many problem arenas. Fig. 1 shows a diagram of a production-inventory system under combined feedback-feedforward control action. Two production nodes are represented by two separate pipes, while the inventory component consists of fluid in a tank. The goal is to manipulate the inflow to the production nodes (i.e., starts) in order to replenish an inventory that satisfies exogeneous demand. The demand signal can be broken down into forecasted and unforecasted components. A substantial literature exists that examines production-inventory systems from a control engineering standpoint (Schwartz and Rivera, 2010). Schwartz and Rivera (2010) examine both Internal Model Control (IMC) and Model Predictive Control (MPC) for a linear production-inventory system with continuous inputs. The hybrid production-inventory system, in which production occurs at discrete levels (or is decided by discrete-event decisions) is an important yet less studied problem; we consider it the focus of this paper.

The paper highlights two application areas that map into the hybrid production-inventory problem space. The first is adaptive interventions in behavioral health, which is a topic receiving increasing attention as a means to address the prevention and treatment of chronic, relapsing disorders, such as drug abuse (Collins et al., 2004). In an adaptive intervention, dosages of intervention components (such as frequency of counseling visits or medication) are assigned to participants based on the values of tailoring variables that reflect some measure of outcome or adherence. Recent work has shown the relationship between forms of adaptive interventions and feedback control of production-inventory systems (Rivera et al., 2007). In prac-

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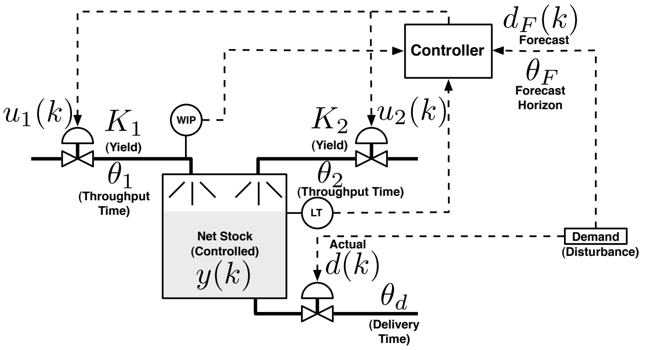

Fig. 1. Diagrammatic representation of a production-inventory system consisting of two production nodes (one primary, one secondary) and a single inventory, with combined feedback-feedforward control.

tice, these problems are hybrid in nature because dosages of intervention components correspond to discrete values. These interventions have to be implemented on a participant population that may display significant levels of interindividual variability. Hence a novel problem formulation is necessary that insures that the decision policy makes appropriate decisions for all members of a population, without demanding excessive modeling effort for each individual participant.

Inventory management in supply chains also represents a rich application area for hybrid production-inventory systems. Consider a scenario where an enterprise is required to make the decisions on startup / shutdown of an auxiliary manufacturing facility in order to achieve operational goals under changing market demand. These problems are hybrid in nature, and the dynamics of these systems can be highly uncertain (Wang and Rivera, 2008). This necessitates a novel problem formulation to insure efficient use of resources and an optimal operating policy for the supply chain management problem, while taking account of the changing market demand.

The paper is organized as follows: Section 2 summarize the MPC formulation with the multi-degree-of-freedom tuning for MLD systems. Two case studies, namely a hypothetical adaptive intervention based on *Fast Track* program, and the supply chain management problem described previously are discussed in Section 3. Summary, conclusions, and directions for future research are presented in Section 4.

2. MODEL PREDICTIVE CONTROL FOR HYBRID SYSTEMS

Model predictive control (MPC) is widely accepted in the process industries due to its ability to systematically include constraints, the capability to handle plants with multiple inputs and outputs, the flexibility given to the user to define a cost function, and its disturbance rejection properties.

2.1 Controller Model

The MPC controller presented in this paper relies on the following model framework (Nandola and Rivera, 2010),

$$x(k+1) = Ax(k) + B_1u(k) + B_2\delta(k) + B_3z(k) + B_dd(k) \quad (1)$$

$$y(k+1) = Cx(k+1) + d'(k+1) + \nu(k+1) \quad (2)$$

$$E_5 \geq E_2\delta(k) + E_3z(k) - E_4y(k) - E_1u(k) + E_dd(k) \quad (3)$$

where x and u represent states (both discrete and continuous) and inputs (both discrete and continuous) of the system. y is a vector of outputs, and d, d' and ν represent measured disturbances, unmeasured disturbances and measurement noise signals, respectively. δ and z are discrete and continuous auxiliary variables that are introduced in order to convert logical/discrete decisions into their equivalent linear inequality constraints summarized in (3) (for details, see Bemporad and Morari (1999)). The framework permits the user to include and prioritize constraints, and incorporate heuristic rules in the description of the model. Because disturbances are an inherent part of any process, it is necessary to incorporate these in the controller model that defines the control system. Equations (1)-(3) are the MLD framework shown in (Bemporad and Morari, 1999), which is modified by incorporating measured and unmeasured disturbances. The model lumps the effect of all unmeasured disturbances on the outputs only, which is a common practice in the process control literature (Wang and Rivera, 2008; Lee and Yu, 1994). We consider d', the unmeasured disturbance, as a stochastic signal, described as follows,

$$x_w(k+1) = A_wx_w(k) + B_ww(k),\ d'(k+1) = C_wx_w(k+1) \quad (4)$$

where A_w has all eigenvalues inside the unit circle and $w(k)$ is a vector of integrated white noise. Here, it is assumed that the disturbance effect is uncorrelated. Thus, $B_w = C_w = I$ and $A_w = diag\{\alpha_1,\ \alpha_1,\ \cdots,\ \alpha_{n_y}\}$ where n_y is number of outputs. In order to take advantage of well understood properties of white noise signal considering difference form of disturbance and system models and augmenting them as follows,

$$X(k+1) = \mathscr{A}X(k) + \mathscr{B}_1\Delta u(k) + \mathscr{B}_2\Delta\delta(k) + \mathscr{B}_3\Delta z(k) + \mathscr{B}_d\Delta d(k) + \mathscr{B}_w\Delta w(k) \quad (5)$$

$$y(k+1) = \mathscr{C}X(k+1) + \nu(k+1) \quad (6)$$

Here $X(k) = [\Delta x^T(k) \quad \Delta x_w^T(k) \quad y^T(k)]^T$, $\Delta * (k) = *(k) - *(k-1)$ and $\Delta w(k)$ is white noise sequence. Augmented matrices $\mathscr{A}$, $\mathscr{B}_i$ and $\mathscr{C}$ are given in Nandola and Rivera (2010).

2.2 MPC Problem

In this work, we use a quadratic cost function of the form,

$$\min_{\{[u(k+i)]_{i=0}^{m-1},\,[\delta(k+i)]_{i=0}^{p-1},\,[z(k+i)]_{i=0}^{p-1}\}} J \triangleq \sum_{i=1}^{p} \|Q_y(y(k+i)-y_r)\|_2^2 + \sum_{i=0}^{m-1} \|Q_{\Delta u}(\Delta u(k+i))\|_2^2 + \sum_{i=0}^{m-1} \|Q_u(u(k+i)-u_r)\|_2^2 + \sum_{i=0}^{p-1} \|Q_d(\delta(k+i)-\delta_r)\|_2^2 + \sum_{i=0}^{p-1} \|Q_z(z(k+i)-z_r)\|_2^2 \quad (7)$$

subjected to mixed integer constraints of (3) and various process and safety constraints such as bound constraints on inputs, outputs and move. Here p is the prediction horizon, m is the control horizon, $(\cdot)_r$ stands for reference trajectory and $\|\cdot\|_2$ is for 2-norm. Q_y, $Q_{\Delta u}$, Q_u, Q_d, and Q_z are penalty weights on the control error, move size, control signal, auxiliary binary variables and auxiliary continuous variables, respectively.

The aforementioned MPC problem is governed by both binary and continuous decision variables hence it is a mixed integer quadratic program (*miqp*). Moreover, it requires future predictions of the outputs and the mixed integer constraints in (3), which can be obtain by propagating (3), (5) and (6) for p steps in future. These multi-step predictions are then used to convert aforementioned MPC problem to a standard *miqp* (for details, see Nandola and Rivera (2010)). This problem can be solved using any *miqp* solver available in the market. In this work, we have used the *Tomlab-CPLEX* solver. It should be noted that the algorithm also requires externally generated reference trajectories, an externally generated forecast of the measured disturbance and estimate of (disturbance free) initial states $X(k)$ that influence the robust performance of the MPC.

The output reference trajectory is generated using an asymptotically step (a Type-I filter per Morari and Zafiriou (1989)) as:

$$\frac{y_r(k+i)}{y_{target}} = \frac{(1-\alpha_r^j)q}{q-\alpha_r^j},\ 1 \le j \le n_y,\ 1 \le i \le p \tag{8}$$

The setpoint tracking speed can be adjusted by choosing α_r^j between [0,1) for each output. The smaller the value, the faster the response for particular setpoint tracking. Thus, setpoint tracking speed can be adjusted for each output individually.

The speed required to reject each measured disturbance can be adjusted independently by using a filter, $f(q,\alpha_d^j)$, $1 \le j \le n_{dist}$, for each measured disturbance. Here n_{dist} is the number of measured disturbances and α_d^j is a tuning parameter between [0,1), for the jth measured disturbance. Smaller the α_d^j, faster the speed of particular disturbance rejection. Thus, filtered signal can be used as an anticipated unmeasured disturbance. The transfer function $f(q,\alpha_d^j)$ can be assumed as an asymptotically step or an asymptotically ramp (*i.e.* Type-I or Type-II signal as per Morari and Zafiriou (1989)), as per the nature of the system dynamics. Type-I filter structure is given in (8) and Type-II filter structure can be given as

$$f(q,\alpha_d^j) = (\beta_0 + \beta_1 q^{-1} + \cdots + \beta_\omega q^{-\omega}) \times \frac{(1-\alpha_d^j)q}{q-\alpha_d^j} \tag{9}$$

$$\beta_k = \frac{-6k\alpha_d^j}{(1-\alpha_d^j)\omega(\omega+1)(2\omega+1)},\ 1 \le k \le \omega \tag{10}$$

$$\beta_0 = 1 - (\beta_1 + \cdots + \beta_\omega) \tag{11}$$

The states of the system can be estimated from the current measurements, $y(k)$ while rejecting the unmeasured disturbance using a Kalman filter as follows:

$$X(k|k-1) = \mathscr{A}X(k-1|k-1) + \mathscr{B}_1\Delta u(k-1) + \mathscr{B}_2\Delta\delta(k-1) + \mathscr{B}_3\Delta z(k-1) + \mathscr{B}_d\Delta d(k-1) \tag{12}$$

$$X(k|k) = X(k|k-1) + K_f(y(k) - \mathscr{C}X(k|k-1)) \tag{13}$$

Here K_f is the filter gain, an optimal value of which can be found by solving an algebraic Riccati equation. We use the parametrization of filter gain (Lee and Yu, 1994) as follows,

$$K_f = [0\ \ F_b\ \ F_a]^T \tag{14}$$

Here $F_a = \text{diag}\{(f_a)_1, \cdots, (f_a)_{n_y}\}$, $F_b = \text{diag}\{(f_b)_1, \cdots, (f_b)_{n_y}\}$, $(f_b)_j = ((f_a)_j{}^2)\backslash(1+\alpha_j - \alpha_j(f_a)_j)$, $1 \le j \le n_y$ and $(f_a)_j$ is a tuning parameter between 0 and 1. While the unmeasured disturbances are rejected using the state observer presented in (12)-(14), the speed of rejection is proportional to the tuning parameter $(f_a)_j$. As $(f_a)_j$ approaches zero, the state estimator increasingly ignores the prediction error correction, and the control solution is mainly determined by the deterministic model, (12). On the other hand, the state estimator tries to compensate for all prediction error as $(f_a)_j$ approaches to 1, with a corresponding increase in the aggressiveness of the control action. In practice, the judicious selection of $(f_a)_j$ requires making the proper tradeoff between performance and robustness.

3. CASE STUDIES

In this section, we present applications of MPC with 3-DoF algorithm on case studies from two different non-traditional areas described in the Introduction: adaptive behavioral interventions and supply chain management.

3.1 Adaptive Time-Varying Behavioral Interventions

As a representative case study of a time-varying adaptive behavioral intervention we examine the hypothetical problem inspired by the *Fast Track* program (C.P.P. Res. Grp., 1992). *Fast Track* was a multi-year, multi-component program designed to prevent conduct disorder in at-risk children. Youth showing conduct disorder are at increased risk for incarceration, injury, depression, substance abuse, and death by homicide or suicide. In *Fast Track*, some intervention components were delivered universally to all participants, while other specialized components were delivered adaptively.

In this paper we analyze a hypothetical adaptive intervention inspired by *Fast Track* for assigning home-based family counseling visits, which are provided to families on the basis of parental functioning. There are several possible levels of intensity, or doses, of family counseling. The idea is to vary the doses of family counseling depending on the needs of the family, in order to avoid providing an insufficient amount of counseling for very troubled families, or wasting counseling resources on families that may not need them or be stigmatized by excessive counseling. The decision about which dose of counseling to offer each family is based primarily on the family's level of functioning, assessed by a family functioning questionnaire completed by the parents.

As described in Collins et al. (2004), families with very poor functioning are given weekly counseling; families with poor functioning are given biweekly counseling; families with near threshold functioning are given monthly counseling; and families at or above threshold are given no counseling. Family functioning is reassessed at a review interval of three months, at which time the intervention dosage may change. This goes on for three years. We consider the scenario in which the total number of home based family counseling visits is limited due to resource constraints, and a monthly group meeting option is made available to participant families once the maximum number of home-based family counseling visits has been reached.

Following as in Rivera et al. (2007), the open-loop dynamics of the intervention is modeled by means of a fluid analogy represented in Fig. 1. Parental function $y_1(k)$ is treated as fluid in a tank , which is depleted by exogenous disturbances $d(k)$. The tank is replenished by the intervention components $u_1(k)$ (home based counseling visits) or $u_2(k)$ (monthly group meeting) , which are manipulated variables. The use of fluid analogy enables developing a mathematical model of the open-loop dynamics of the intervention using the principle of conservation of mass. This model can be described by following difference equations which relates parental function $y_1(k)$ with the intervention components $u_1(k)$ and $u_2(k)$ as follows:

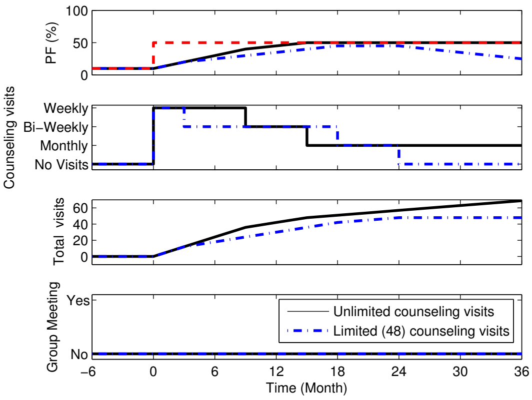

Fig. 2. Adaptive behavioral intervention case study. Control performance for unlimited and limited (48) counseling visits, in the absence of the group meeting component. Step disturbance $D(k) = 5$, $\alpha_r = 0$, $\alpha_d = 0.$, $f_a = 1$, $Q_y = 1$, $Q_{\Delta u} = 0.05$, $Q_u = Q_d = Q_z = 0$.

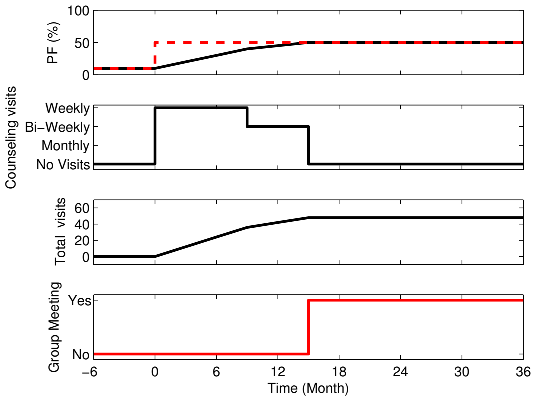

Fig. 3. Adaptive behavioral intervention case study. Control performance for limited (48) counseling visits, with a group meeting component available. Step disturbance $D(k) = 5$, $\alpha_r = \alpha_d = 0.$, $f_a = 1$, $Q_y = 1$, $Q_{\Delta u} = 0.05$, $Q_u = Q_d = Q_z = 0$.

$$y_1(k+1) = y_1(k) + K_1 u_1(k - \theta_1') + K_2 u_2(k - \theta_2') - d(k) \quad (15)$$

$K_1 = 0.15$ and $K_2 = 5$ represent the intervention gain for counselor home visits and monthly group meeting, respectively, $\theta_1' = \theta_1 - 1 = 0$ represents the time delay between the intervention component u_1 and its effect on parental function, $\theta_2' = \theta_2 - 1 = 0$ represents the time delay between the intervention component u_2 and its effect on parental function. $d(k)$ is the unknown source of parental function depletion (*i.e.* unmeasured disturbance). The measured disturbance and hence feedforward control is not considered here in this application.

Here it should be noted that $u_1(k)$ has a restriction on the frequency of counselor visits such that these can be only possible either weekly, biweekly or monthly, or none at all. This problem requires imposing a restriction on the intervention $u_1(k)$ such that it takes only four values: 0, u^{weekly}, $u^{biweekly}$ and $u^{monthly}$. In order to capture discreteness in the intervention, four binary auxiliary variables, δ_1, δ_2, δ_3, δ_4 and three continuous auxiliary variables, z_1, z_2, z_3 are introduced. The detailed description of logical conditions are not presented here for the brevity of the paper. Moreover, $u_2(k)$ has a restriction such that it can be available *if and only if* intervention component of home based counseling visits is exhausted (*i.e.* total 48 home visits completed). Thus, u_1 and u_2 can not be given simultaneously. In order to implement this restriction one additional binary variable δ_5 is introduced with the following additional relationships,

$$y_2(k) = y_2(k-1) + b_1 z_1(k) + b_2 z_2(k) + b_3 z_3(k) \quad (16)$$

$$\delta_5(k) = 1 \Leftrightarrow 48 - y_2(k) \;\&\; u_2(k) = \delta_5(k) \quad (17)$$

Here y_2 refers total number of counselor home visits. Equation (17) make sure that intervention component corresponding to monthly group meeting is available only in a situation where home based counselor visit is not available. Thus, the problem has inherent discreteness along with the continuous dynamics, which can be characterized by the hybrid dynamical system and can be model using the MLD framework.

Control performance for unlimited number of counseling visits in absence of group meeting (*i.e.* nominal case) is documented in Fig. 2 using solid line. From the figure, it can be seen that the parental function achieves desired goal satisfactorily using 69 in-home counseling visits during the period of 3 years. While dashed-dotted line in Fig. 2 represent control performance for the case where total home based counseling visits are restricted to 48 without availability of the group meeting (*i.e.* $K_2 = 0$). From the figure, it is observed that the dosage constraints places a fundamental limit on the effects of the intervention and it is unable to achieve desired parental function goal in absence of any other additional intervention. However, as it is clearly seen from the figure, the controller does the best that it can and uses available resources such that it can reach as close as possible to the the desired goal. A decline in the parental function after 24 months is noticed because of the depletion of the parental function by some unknown disturbances and at the same time unavailability of the intervention dosage (*i.e* in-home counseling visits). Fig. 3 documents performance for the case with similar restriction on total number of visits. However, in this case, option for the monthly group meeting is available (*i.e.* $K_2 = 5$) on completion of 48 home based counseling visits. Here it can be seen that the decision policy assigns intervention dosages (in-home counseling visits) similar to the nominal case until total number of visits reach to maximum value of 48 and then it assigns group meeting in order to reach and maintain the desired parental function goal. Thus, the proposed MPC-based decision policy is capable of enforcing constraints on intervention dosages, as well as switching between two different interventions as needed.

3.2 Supply Chain Management Case Study

In this case study, we consider a supply chain management problem comprised of a production-inventory system with one inventory and two production nodes. The production nodes consist of one primary factory and a secondary auxiliary one, as shown in Figure 1. Throughput time $\theta_1 = \theta_1' + 1$ and yield K_1 for the primary factory are 4 days and 0.9, respectively; for the auxiliary factory, the throughput time $\theta_2 = \theta_2' + 1$ and yield K_2 are 9 days and 0.8, respectively. Here we consider that the operating cost for the auxiliary factory is greater than the primary one because of the longer throughput time and lower yield. Therefore, this auxiliary production node should be accessed *if and only if* the primary node is running at its full capacity and unable to meet future market demand; likewise, production from the auxiliary node must be discontinued if future demand cannot justify its operation. The discrete decision of startup or shutdown of the auxiliary factory is a function of the continuous variables $d_f(k)$ (demand forecast) and the work-in-progress (WIP) in the primary production node. Consequently, the system can be categorized as a hybrid system, where the inventory dynamics are governed by continuous variables (inventory, factory starts, demand and WIP) and a discrete decision (shutdown/start-up of the auxiliary factory). The dynamics of this system can be described using first principles model follows:

$$y(k+1) = y(k) + K_1 u_1(k-\theta_1') + K_2 u_2(k-\theta_2') - d(k) \quad (18)$$

$$WIP(k+1) = \sum_{i=0}^{\theta_1'} u_1(k-i) \quad (19)$$

where $d(k) = d_f(k-\theta_f) + d_u(k)$ is the total customer demand comprising forecasted d_f and unforecasted d_u components, $u_1(k) \in [0,\ 200]$ represents the starts for the primary production node, $u_2(k) \in [0,\ 200]$ is the starts for the auxiliary production node, and $WIP \in [0,\ 600]$ represents the work-in-progress in the primary production node. In order to ensure that the auxiliary factory is activated *if and only if* WIP in the primary factory is at its maximum capacity, we embed the following logical condition $u_2(k-\theta_2') \neq 0 \Leftrightarrow WIP(k+1) > Cap_{max} = 600$ into the dynamical model. The implication ($\Leftrightarrow$) can be converted into linear inequality constraints using Big-M constraints (Raman and Grossmann, 1991). This conversion can be accomplished by introducing an auxiliary binary variable (δ) and an auxiliary continuous variable (z) followed by a MLD representation of (18)-(19) as in (1)-(3). The MLD model of (18)-(19) and linear constraint from aforementioned logical condition along with bound constraints on process variables are then used to formulate an MPC problem. The operational goal of this system is to meet the customer demand $d(k)$ relying on production from the auxiliary factory only when necessary, while maintaining the net stock inventory level at a predefined setpoint. This can be accomplished by manipulating factory starts $u_1(k)$, $u_2(k)$ and feedforward compensation of forecasted demand $d_f(k)$ simultaneously imposing the above described logical condition. Here we consider a sampling interval of $T = 1$ day, $\theta_f = p = 30$ days.

In practice, it is desirable to keep the factory starts as constant as possible (i.e., avoid factory "thrash") while maintaining inventory at desired levels in the face of uncertain customer demand (Wang and Rivera, 2008). In order to examine the performance of the proposed multi-degree-of-freedom MPC formulation, we consider a piecewise stochastic customer demand signal and its forecast, which are shown in Figure 4. Figure 5 demonstrates the performance of the proposed formulation that uses multi-degree-of-freedom tuning parameters, $\alpha_r = 0.9$, $\alpha_d = 0$, $f_a = 0.1$, penalty weight parameters $Q_y = 1$, $Q_{\Delta u} = diag\{0\,0\}$, $Q_u = Q_d = Q_z = 0$, prediction horizon $p = 30$ and control horizon $m = 25$. From the figure, it can be seen that the proposed MPC algorithm is able to satisfactorily maintain inventory at its predefined target, while producing little variation in the starts of the factories. Moreover, it is able to decide on the start-up / shutdown of the auxiliary factory in a desirable manner (i.e., by making minimal use of the auxiliary factory). This reduces operational cost while allowing the production system to meet customer demand without backorders.

In order to assess the effectiveness of the proposed formulation, we compare its performance with the MPC formulation that relies on a constant plant-model mismatch over the prediction horizon. This formulation uses the move suppression weight ($Q_{\Delta u}$) in lieu of f_a to reduce the variation in the manipulated

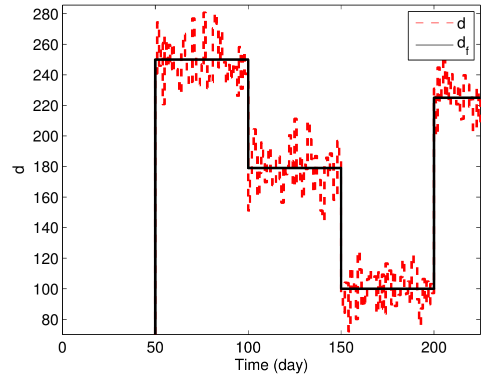

Fig. 4. Customer demand $d(k)$ (red dashed line) and demand forecast $d_f(k)$ (solid black line), SCM case study.

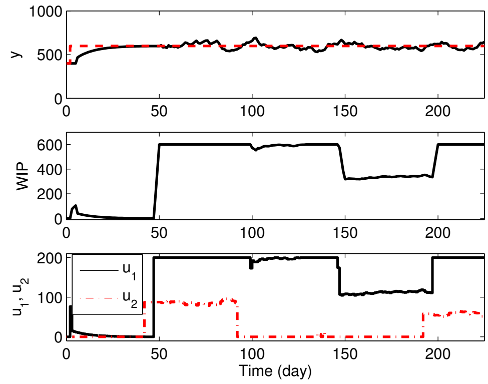

Fig. 5. Response of net stock (y), work-in-progress (WIP), and factory starts (u_1,u_2) for the proposed multiple-degree-freedom formulation with tuning parameters: $\alpha_r = 0.9$, $\alpha_d = 0$, $f_a = 0.1$ and $Q_y = 1$, $Q_{\Delta u} = diag\{0\ 0\}$, $Q_u = Q_d = Q_z = 0$ for the demand in Fig. 4.

variables (i.e., starts) of the factories. Figure 6 presents the simulation results using $Q_{\Delta u} = diag\{0\ 0\}$ while keeping all other parameters as in the previous case. The responses show evidence of very poor inventory control and very high variation in the starts of the factories as compared to the proposed formulation. To reduce the variation in the starts of the factories and verify the performance against the proposed formulation, we apply various values of the move suppression weight $Q_{\Delta u}$. Table 1 documents the maximum (peak) value of the inventory (y_{max}) and the closed-loop performance metrics J_e and $J_{\Delta u}$ using the MPC formulation relying on a constant plant-model mismatch over the prediction horizon for five values of $Q_{\Delta u}$ between 0 to 200. The last row of Table 1 documents these values for the three-degree-of-freedom (3-DoF) MPC formulation. The performance metrics J_e and $J_{\Delta u}$ are measures of cumulative error ($e(k) = y(k) - y_r$) and variation in the rate-of-change of factory starts ($\Delta u(k) = u(k) - u(k-1)$), respectively, which are given as $J_* = \sum_{k=1}^{t/T_s} *(k)^T * (k)$, where $* = e,\ \Delta u$.

From the table, it can be seen that the proposed 3-DoF formulation outperforms the constant plant-model mismatch based MPC formulation in terms of maximum peak in the inventory and J_e for all values of $Q_{\Delta u}$. On the other hand, increasing the move suppression weight $Q_{\Delta u}$ lowers $J_{\Delta u}$, with the last two cases (*i.e.* $Q_{\Delta u} = diag\{100\ 100\}$ and $Q_{\Delta u} = diag\{200\ 200\}$) yielding lower values of $J_{\Delta u}$ than the 3-DoF MPC formulation.

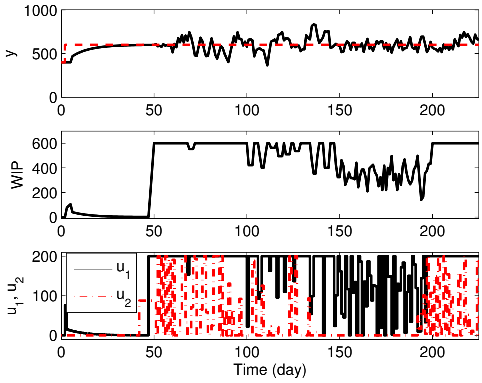

Fig. 6. Response of net stock (y), Work-in-Progress (WIP), and factory starts (u_1,u_2) for the MPC formulation that relies on constant plant-model mismatch and move suppression tuning, with $Q_{\Delta u} = diag\{0\,0\}$ and $Q_y = 1$, $Q_u = Q_d = Q_z = 0$ for the demand profile in Fig. 4.

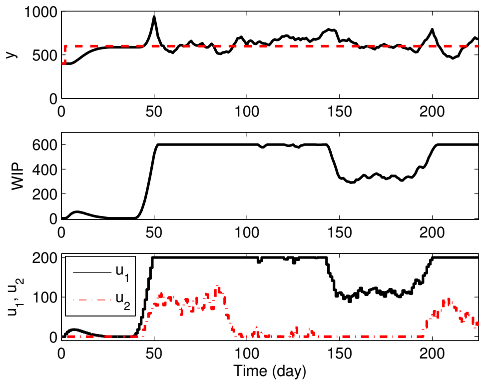

Fig. 7. Response of net stock (y), Work-in-Progress (WIP), and factory starts (u_1,u_2) for the MPC formulation that relies on constant plant-model mismatch and move suppression tuning with $Q_{\Delta u} = diag\{200\,200\}$ and $Q_y = 1$, $Q_u = Q_d = Q_z = 0$ for the demand profile in Fig. 4.

Table 1. $J_{\Delta u}$, J_e and y_{max} using MPC relying on constant plant-model mismatch for various $Q_{\Delta u}$ and a 3-DoF MPC formulation for $f_a = 0.1$.

Sl. No.	$Q_{\Delta u}$	$J_{\Delta u}$	J_e	y_{max}
1	$diag\{0\,0\}$	267.9×10^4	13.15×10^5	834.63
2	$diag\{1\,1\}$	94.67×10^4	11.15×10^5	763.865
3	$diag\{10\,10\}$	25.82×10^4	10.69×10^5	809.24
4	$diag\{100\,100\}$	4.81×10^4	12.71×10^5	890.94
5	$diag\{200\,200\}$	3.038×10^4	15.71×10^5	939.24
6	3-DoF	8.345×10^4	4.426×10^5	692.76

Contrasting Fig. 5 with Fig. 7 shows that for the 3-DoF formulation, factory starts remain constant over a significant portion of the simulation, without leading to the substantial maximum inventory peak and corresponding demands on warehouse space resulting from $Q_{\Delta u} = diag\{200\,200\}$. Thus, we observe that the 3-DoF-MPC algorithm can be useful for reducing overall operating costs and efficiently managing this class of hybrid production-inventory systems.

4. SUMMARY

Control applications of hybrid systems are becoming increasingly important in many fields, among them process control. In this work, the application of hybrid model predictive control (HMPC) to adaptive time-varying behavioral interventions and inventory management in supply chains, two problems that can be conceptualized using fluid analogies, are presented. These systems are represented as mixed logical dynamical (MLD) models, which can then be used to specify a hybrid model predictive control law relying on the three-degree-of-freedom (3-DoF) tuning formulation of Nandola and Rivera (2010). The use of 3-DoF tuning enables individually adjusting the speed of disturbance rejection (measured and unmeasured) and setpoint tracking for each output; this has intuitive appeal for the user and facilitates achieving robust operation under uncertainty. The effectiveness of the proposed formulation is demonstrated in these applications under diverse scenarios involving variations in constraints, disturbances, and tuning.

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