

Hybrid Model Predictive Control for Optimizing Gestational Weight Gain Behavioral Interventions

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Abstract—Excessive gestational weight gain (GWG) represents a major public health issue. In this paper, we pursue a control engineering approach to the problem by applying model predictive control (MPC) algorithms to act as decision policies in the intervention for assigning optimal intervention dosages. The intervention components consist of education, behavioral modification and active learning. The categorical nature of the intervention dosage assignment problem dictates the need for hybrid model predictive control (HMPC) schemes, ultimately leading to improved outcomes. The goal is to design a controller that generates an intervention dosage sequence which improves a participant’s healthy eating behavior and physical activity to better control GWG. An improved formulation of self-regulation is also presented through the use of Internal Model Control (IMC), allowing greater flexibility in describing self-regulatory behavior. Simulation results illustrate the basic workings of the model and demonstrate the benefits of hybrid predictive control for optimized GWG adaptive interventions.

I. INTRODUCTION

High pre-pregnancy body mass index (BMI) and excessive gestational weight gain (GWG) have become increasingly important public health concerns. Among pregnant women, the prevalence of overweight and obesity ($\text{BMI} \geq 25 \text{ kg/m}^2$ and $\text{BMI} \geq 30 \text{ kg/m}^2$ respectively) has almost doubled in the last 20 years, from 29.7% in 1983 to 53.7% in 2011, with almost one half of pregnant women in the United States now beginning their pregnancies as either overweight or obese [1]. Many studies have found that excessive weight gain during pregnancy is often associated with maternal obesity post-partum and a number of adverse pregnancy outcomes, such as gestational diabetes mellitus, pregnancy-related hypertension, complications through labor and delivery, and macrosomia [2], [3]. Excessive GWG is also a potential prenatal risk factor for childhood obesity [4].

In 2009, the US Institute of Medicine (IOM) published revised GWG guidelines for how much weight a woman should

gain during pregnancy to optimize both maternal and child outcomes; these highlight the importance of intervention during pregnancy to prevent both maternal post-partum obesity and childhood obesity [3]. Though traditional interventions appear to reduce the risk of adverse pregnancy outcomes among normal weight women [5], [6], these have been less effective for overweight and obese women. Therefore, there is a critical need to develop scalable, effective and affordable interventions to help overweight and obese pregnant women prevent high GWG. Adaptive interventions [7] represent a promising approach to prevention and treatment in this regard. In an adaptive intervention, dosages of intervention components are assigned to participants based on values of tailoring variables. The use of dynamical systems and control engineering methods to optimize adaptive behavioral interventions is described in [8], [9].

In prior work [10], a comprehensive dynamical model for a behavioral intervention to control GWG was proposed. This work demonstrated how to integrate a mechanistic energy balance (EB) model for GWG with a dynamical behavioral model incorporating principles from behavioral (Theory of Planned Behavior) and self-regulatory theories. This paper extends prior work through the design and implementation of an optimized behavioral intervention based on hybrid model predictive control (HMPC). Hybrid systems are dynamical systems that involve the interaction between continuous and discrete dynamics. The term hybrid has also been applied to describe processes that involve continuous dynamics and discrete (logical) decisions [8], [11].

This work demonstrates the application of HMPC as a discrete decision framework for a behavioral intervention which can be conceptualized in terms of a production-inventory system. This form of adaptive behavioral intervention features multiple intervention components, requiring behavioral scientists to formulate and evaluate decision rules that dictate the proper dosage sequence, specifying the order in which each component should be augmented or reduced. The problem is well-suited for a hybrid predictive control solution. The proposed scheme relies on the use of Mixed Logical Dynamical (MLD) framework for the control of hybrid systems developed in [11], and implements the improved three degree-of-freedom (DoF) tuning formulation of Nandola and Rivera [8]. The novelty of this paper lies in the development of a controller that both assigns the discrete dosages and also generates the proper dosage sequence, and its application to a GWG model that takes into account EB as well as established psychological principles.

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TABLE I: Target gestational weight gain (GWG) recommended by the 2009 Institute of Medicine guidelines [3] and the corresponding trimester-specific energy intake that coincides with this maternal weight gain [12].

Classification	Pre-gravid BMI (kg/m ²)	Target GWG (kg) Trimester		Model Predicted ΔEI (kcal/d) Trimester		
		1	2 - 3	1	2	3
Underweight	<20	0.5 - 2.0	11.4 - 15.8	94 - 184	400 - 511	442 - 574
Normal	20 - 25	0.5 - 2.0	9.1 - 13.0	94 - 200	381 - 492	444 - 635
						269 - 364
						227 - 326

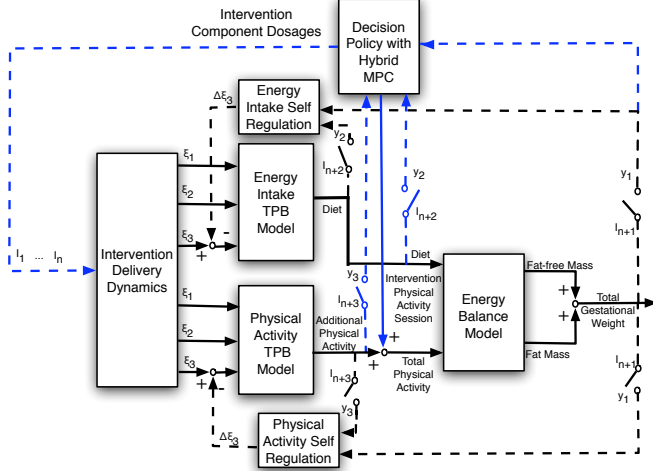


Fig. 1: Schematic representation for an “adaptive”/optimized gestational weight gain (GWG) intervention by HMPC.

Fig. 1 shows the overall schematic representation for GWG intervention in this paper, which can be divided into five main segments: a two-compartment EB model, two TPB models, an intervention delivery module, two self-regulation modules and the decision policy that is operated by the HMPC to optimize the behavioral intervention and improve the participant response. The overall model serves a useful role in the generation of decision policies in an adaptive intervention by the use of advanced control strategies.

This paper is organized as follows. Section II presents a modeling overview extended from our earlier work in [10], including a brief description of the EB model, the fluid analogy for TPB, the behavioral intervention inputs, and an improved formulation of self-regulation modeling based on Internal Model Control (IMC) allowing greater flexibility in the response. Section III focuses on the definition of decision framework in which HMPC is applied to the intervention model. Section IV discusses the simulation results for a case study based on hypothetical parameters. Section V gives a summary of our conclusions and future work.

II. MODELING OVERVIEW

A. Energy Balance Model

The EB model for GWG is based on the two-compartment model developed by Thomas *et al.* [12]. It relies on the conservation of the energy, which can be expressed as,

$$ES(t) = EI(t) - EE(t) \quad (1)$$

$ES(t)$ is the energy stored, $EI(t)$ is the energy intake and $EE(t)$ is the energy expenditure at time t , measured daily.

The total body mass (BM) corresponds to the sum of the two compartments: fat mass (FM) and fat-free mass (FFM).

$$BM(t) = FM(t) + FFM(t) \quad (2)$$

The ES term can be expanded into the sum of the instantaneous change of the two compartments (FFM and FM), multiplied by their respective energy densities. If the increase of energy intake (EI) and physical activity (PA) are included in (1) as two inputs, it gives,

$$771 \frac{dFFM}{dt} + 9500 \frac{dFM}{dt} = (1 - g)(EI_0 + \Delta EI(t)) - (1 + \Delta PA(t))EE(t) \quad (3)$$

where g is the nutrient partitioning constant, EI_0 is the initial EI, ΔEI and ΔPA are the two inputs.

B. Fluid Analogy for Theory of Planned Behavior (TPB)

The TPB [13] is a general social-cognitive theory that can be used to describe the behavioral component of human weight change interventions. A dynamic TPB model can be postulated as a fluid analogy [14] consisting of five inventories where each component of the TPB is represented by an inventory, as depicted in Fig. 2, with inflows corresponding to the exogenous variables ξ_1 , ξ_2 , and ξ_3 . The dynamical system description can be generated when the principle of conservation of mass is applied to each inventory,

$$\tau_1 \frac{d\eta_1}{dt} = \gamma_{11}\xi_1(t - \theta_1) - \eta_1(t) + \zeta_1(t) \quad (4)$$

$$\tau_2 \frac{d\eta_2}{dt} = \gamma_{22}\xi_2(t - \theta_2) - \eta_2(t) + \zeta_2(t) \quad (5)$$

$$\tau_3 \frac{d\eta_3}{dt} = \gamma_{33}\xi_3(t - \theta_3) - \eta_3(t) + \zeta_3(t) \quad (6)$$

$$\tau_4 \frac{d\eta_4}{dt} = \beta_{41}\eta_1(t - \theta_4) + \beta_{42}\eta_2(t - \theta_5) + \beta_{43}\eta_3(t - \theta_6) - \eta_4(t) + \zeta_4(t) \quad (7)$$

$$\tau_5 \frac{d\eta_5}{dt} = \beta_{54}\eta_4(t - \theta_7) + \beta_{53}\eta_3(t - \theta_8) - \eta_5(t) + \zeta_5(t) \quad (8)$$

τ_i are time constants, θ_i time delays, ζ_i disturbances, and β_{ij} and γ_{ij} gains of the system.

C. Intervention Programs

This paper uses the most effective intervention for GWG which combines healthy eating (HE) and PA [15] to help pregnant women meet the IOM guidelines (Table I). The proposed intervention components are informed by past research and experience [16], and include GWG, HE and PA education, and behavioral modification. Evidence shows that these components can effectively manage weight, and when people are taught how to use these strategies, they

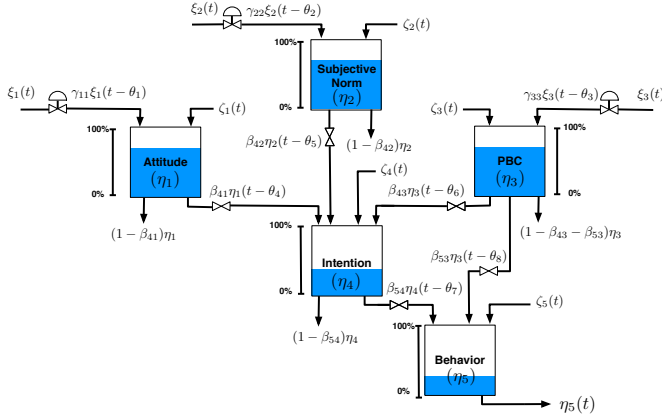


Fig. 2: Fluid Analogy for the TPB.

are more likely to achieve their goals and positively impact outcomes [17]. [18]–[20] show that active participation in strategies to promote HE is effective for increasing weight loss. Participants who engage in PA behavior modification (e.g., engage in moderate-intensity PA behaviors) have higher attitude, PBC, intention and subjective norm compared to those who are not actively engaged in these behaviors [17].

The modeling of the intervention delivery dynamics is treated like a production inventory system. (9) shows the new PBC inflow to the energy intake TPB (EI-TPB) model under the influence of n intervention components,

$$\xi_3^E(t) = \xi_{3b}^E + \int_{t_0}^{t_f} \left[\sum_{i=1}^n k_{3i} I_i(t - \theta_{3i}^E) \right] dt + \xi_3^E(t) \quad (9)$$

where I_i is the dosage of intervention components; k_{3i} correspond to intervention gains. The expressions for the other inflows to the TPB models with the intervention effect are obtained similarly.

D. Self-Regulation

Self-regulation theory in psychology has been largely influenced by the work of Carver and Scheier [21] who proposed that human behavior is goal-directed and regulated by feedback control processes. Self-regulation is enhanced by

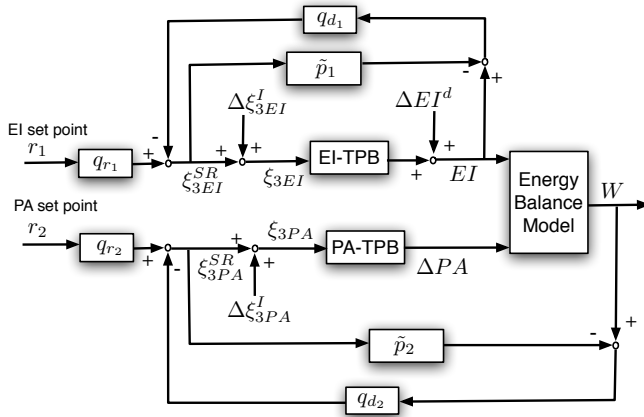


Fig. 3: Closed-loop system implemented with self-regulation designed by 2 DoF IMC.

repeated measurement and assessment of important outcomes in an intervention.

This paper uses two DoF IMC controllers to model self-regulation for EI and PA. This differs from the approach in [10]. Self-regulation is implemented as a controller that adjusts the PBC inflows to the TPB models based on the discrepancies between reference values and measured outcomes. The EI reference values (r_1) and IOM guidelines (r_2) in Table I are the set points for the EI self-regulation and PA self-regulation, respectively (Fig. 3). Assuming no time delay and no plant/model mismatch, the closed-loop expressions for manipulated variables (PBC inflows in EI-TPB and PA-TPB adjusted by self-regulation: ξ_{3EI}^{SR} and ξ_{3PA}^{SR}) and controlled variables (EI and W) are,

$$\xi_{3EI}^{SR} = q_{r1} r_1 - q_{d1} (\Delta EI^d + \tilde{p}_1 \Delta \xi_{3EI}^I) \quad (10)$$

$$EI = \tilde{p}_1 q_{r1} r_1 + (1 - \tilde{p}_1 q_{d1}) (\Delta EI^d + \tilde{p}_1 \Delta \xi_{3EI}^I) \quad (11)$$

$$\xi_{3PA}^{SR} = q_{r2} r_2 - q_{d2} \tilde{p}_2 \Delta \xi_{3PA}^I - q_{d2} \tilde{p}_d [\tilde{p}_1 q_{r1} r_1 + (1 - \tilde{p}_1 q_{d1}) (\Delta EI^d + \tilde{p}_1 \Delta \xi_{3EI}^I)] \quad (12)$$

$$W = \tilde{p}_2 q_{r2} r_2 + (1 - \tilde{p}_2 q_{d2}) [\tilde{p}_d (1 - \tilde{p}_1 q_{d1}) \tilde{p}_1 \Delta \xi_{3EI}^I + \tilde{p}_2 \Delta \xi_{3PA}^I + \tilde{p}_d \tilde{p}_1 q_{r1} r_1 + \tilde{p}_d (1 - \tilde{p}_1 q_{d1}) \Delta EI^d] \quad (13)$$

where \tilde{p}_1 is the EI-TPB model, \tilde{p}_d is the approximated EB model for the EI input, \tilde{p}_2 is the PA-TPB model cascading with the approximated EB model for the PA input; ΔEI^d is the EI increase due to pregnancy (disturbance); $\Delta \xi_{3EI}^I$ and $\Delta \xi_{3PA}^I$ are PBC inflow changes by intervention in EI-TPB and PA-TPB, respectively; q_{r1} , q_{r2} , q_{d1} , and q_{d2} are the controllers designed by 2 DoF IMC and augmented with the filters in the closed loop system to follow the set point changes and reject the disturbances, which are defined as,

$$q_{r1} = \tilde{p}_1^{-1} f_{r1} = \frac{1}{(\lambda_{r1} s + 1)^3} \tilde{p}_1^{-1} \quad (14)$$

$$q_{r2} = \tilde{p}_2^{-1} f_{r2} = \frac{1}{(\lambda_{r2} s + 1)^3} \tilde{p}_2^{-1} \quad (15)$$

$$q_{d1} = \tilde{p}_1^{-1} f_{d1} = \frac{1}{(\lambda_{d1} s + 1)^3} \tilde{p}_1^{-1} \quad (16)$$

$$q_{d2} = \tilde{p}_2^{-1} f_{d2} = \frac{4\lambda_{d2} s + 1}{(\lambda_{d2} s + 1)^4} \tilde{p}_2^{-1} \quad (17)$$

where q_{r1} , q_{r2} , and q_{d1} are augmented with Type I filters because the two set points and EI increase are all step change; q_{d2} is augmented with a Type II filter [22], because the EB model is approximated as two integrators which will be described in the ensuing section. λ_{r1} , λ_{r2} , λ_{d1} and λ_{d2} are the filter parameters, respectively.

III. HYBRID MPC AS A DECISION FRAMEWORK

In an adaptive, time-varying intervention, the frequency or intensity of intervention dosages will change over time, based on the result of important outcomes of the intervention (also known as tailoring variables [7]). Decision rules operationalize these changes, eliminating / adding some intervention components based on participant response during the intervention, or altering the dosage of existing components.

In a control engineering approach to optimizing an adaptive time-varying intervention, the controller assigns dosages

of each intervention component to the participant as dictated by model dynamics, problem constraints, and disturbances. This paper focuses on the application of model predictive control as an algorithmic framework for the decision making to assign these systematic dosages. This control algorithm makes use of feedback and feedforward control action by online optimization of a cost function using a receding horizon and well suited for designing behavioral interventions.

The key point for an adaptive behavioral intervention problem is that they are hybrid in nature because dosages of intervention components correspond to discrete values and the decision is made in a discrete manner; therefore it is necessary to consider hybrid algorithms [8]. The MLD framework is used to represent linear hybrid systems with real and integer states, inputs and constraints [11] as follows,

$$x(k+1) = Ax(k) + B_1u(k) + B_2\delta(k) + B_3z(k) + B_d d(k) \quad (18)$$

$$y(k) = Cx(k) + d'(k) + \nu(k) \quad (19)$$

$$E_2\delta(k) + E_3z(k) \leq E_5 + E_4y(k) + E_1u(k) - E_d d(k) \quad (20)$$

where x and u represent states and inputs of the system (both continuous and discrete). y is the output and d , d' and ν represent measured disturbances, unmeasured disturbances and measurement noise signals, respectively. δ and z are discrete and continuous auxiliary variables that are introduced in order to convert logical/discrete decisions into their equivalent linear inequality constraints. This framework permits the user to include and prioritize constraints, and the general rules in the description of the model is described here. Details of the controller formulation are found in [8].

In this work, we rely on a three DoF approach to tune the controller that allows the user to adjust the speed of set point tracking, measured and unmeasured disturbance rejection independently [8] in the closed-loop system by varying parameters α_r , α_d and f_a respectively. These parameters can be adjusted between values 0 and 1, and they in turn alter the response of the Type I filter ($f(q, \alpha_r)$) and the Type II filter ($f(q, \alpha_d)$) in [22] as

$$f(q, \alpha_r) = \frac{(1 - \alpha_r)q}{q - \alpha_r} \quad (21)$$

$$f(q, \alpha_d) = (\beta_0 + \beta_1q^{-1} + \dots + \beta_\omega q^{-\omega}) \times \frac{(1 - \alpha_d)q}{q - \alpha_d} \quad (22)$$

$$\beta_k = \frac{-6k\alpha_d}{(1 - \alpha_d)\omega(\omega + 1)(2\omega + 1)} \quad (23)$$

$$\beta_0 = 1 - (\beta_1 + \dots + \beta_\omega) \quad (24)$$

which gives a filtered signal to the controller and measured disturbance rejection or adjust the observer gain for unmeasured disturbance rejection as

$$K_f = [0 \quad (f_a)^2 \quad f_a]^T \quad (25)$$

The quadratic cost function used in this paper is,

$$\min_{\{[u(k+i)]_{i=0}^{m-1}, [\delta(k+i)]_{i=0}^{p-1}, [z(k+i)]_{i=0}^{p-1}\}} J \triangleq \sum_{i=1}^p \|y(k+i) - y_r\|_{Q_y}^2 \quad (26)$$

subject to mixed integer constraints according to (20), various process constraints and additional constraints relating the dosage sequence.

$$y_{min} \leq y(k+i) \leq y_{max}, \quad 1 \leq i \leq p \quad (27)$$

$$u_{min} \leq u(k+i) \leq u_{max}, \quad 0 \leq i \leq m-1 \quad (28)$$

$$\Delta u_{min} \leq \Delta u(k+i) \leq \Delta u_{max}, \quad 0 \leq i \leq m-1 \quad (29)$$

where p is the prediction horizon, m the control horizon, y_r the reference, Q_y the penalty weight on the control error.

An approximation of the EB model is used in the design of the H MPC for decision policy and the IMC for self-regulation and is approximated as two integrators,

$$\Delta BM(t) = \frac{K_{EI}}{s} \Delta EI + \frac{K_{PA}}{s} \Delta PA \quad (30)$$

As there are many intervention components (I_1, \dots, I_n), the dosages of which are manipulated variables, we have to consider the priority and sequence of the dosages augmentation and reduction. In this paper, we demonstrate how H MPC works as a decision policy that generates a dosage sequence and assigns discrete intervention dosages. We assume that in the baseline intervention, all the components enter at the same time with a base dosage. There are two intervention components which will be augmented; these are component engaging in HE behavior (I_1) with two augmentations from baseline and component engaging in PA behavior (I_2) with three augmentations from baseline. The system characterized by discrete inputs can be represented logically as:

$$\delta_i(k) = 1 \Leftrightarrow z_i(k) = i; \quad i \in \{0, 1, 2, 3\} \quad (31)$$

$$I_1(k) = \sum_{i=0}^3 z_i(k) \quad \sum_{i=0}^3 \delta_i(k) = 1 \quad (32)$$

$$\delta_j(k) = 1 \Leftrightarrow z_j(k) = j - 4; \quad j \in \{4, 5, 6, 7, 8\} \quad (33)$$

$$I_2(k) = \sum_{j=4}^8 z_j(k) \quad \sum_{j=4}^8 \delta_j(k) = 1 \quad (34)$$

$$\delta_\ell(k) = 1 \Leftrightarrow z_\ell(k) = \ell - 9; \quad \ell \in \{9, 10\} \quad (35)$$

$$I_m(k) = \sum_{\ell=9}^{10} z_\ell(k) \quad \sum_{\ell=9}^{10} \delta_\ell(k) = 1; \quad m \in \{3, \dots, n\} \quad (36)$$

where (31) and (32) describe the dosage change for I_1 , with $I_1 = 1$ as base dosage, $I_1 = 2$ and $I_1 = 3$ as two augmentations from the baseline, and $I_1 = 0$ as the reduction from the baseline; (33) and (34) describe the dosage augmentation and reduction for I_2 in a similar manner; (35) and (36) do so for the other intervention components.

Moreover, the additional constraints on variables are,

$$0 \leq y \leq 25 \quad (37)$$

$$0 \leq I_1 \leq 3 \quad (38)$$

$$0 \leq I_2 \leq 4 \quad (39)$$

$$0 \leq I_m \leq 1; \quad m \in \{3, \dots, n\} \quad (40)$$

$$-1 \leq \Delta I_\ell \leq 1; \quad \ell \in \{1, \dots, n\} \quad (41)$$

The upper bound of output y is determined based on the case when there is neither intervention nor self-regulation.

The move constraint in (41) can guarantee that the dosage augmentation or reduction will take place one step at a time.

The multiple intervention components in this problem requires having to face the decision regarding which component should be first augmented or reduced. I_2 will be augmented only when I_1 reaches full augmentation, while I_1 will be reduced from maximum dosage only when I_2 returns back to the base dosage (augmentation and reduction sequence). When it is necessary to reduce the dosage from the baseline, I_2 is reduced first, followed by I_1 and the other components (reduction sequence). At each assessment cycle, there will be only one intervention component augmented or reduced by one step. Based on the input augmentation and reduction sequence above, the following logical conditions are embedded into the dynamical model (base dosage is 1):

$$I_2(k+T) > 1 \Rightarrow I_1(k) = I_1(k+T) = 3 \quad (42)$$

$$0 < I_1(k+T) < 3 \Rightarrow I_2(k) = I_2(k+T) = 1 \quad (43)$$

$$I_1(k+T) = 0 \Rightarrow I_2(k) = I_2(k+T) = 0 \quad (44)$$

$$I_m(k+T) = 0 \Rightarrow I_1(k) = I_1(k+T) = 0; \\ m \in \{3, \dots, n\} \quad (45)$$

where k is the current time, and T the sampling time.

IV. SIMULATION RESULTS AND DISCUSSION

In this section, we consider a hypothetical simulation scenario illustrating how optimal decision policy is developed based on HMPC. The simulations consider a 25-year old pregnant woman with pre-gravid parameters of height (=1.6m) and weight (=75 kg), which classifies her in the overweight BMI category (BMI=29.30). For the sake of simplicity, we only focus on the effects that intervention components and self-regulation play on the PBC inflow in the TPB models. We assume that the participant will have a ramp increase in her EI from day 14 to day 91 (which is treated as the disturbance in the system) and her EI will keep constant from day 92 to the delivery.

The case study assumes that all the participants enter the intervention at week 14 with base dosage. The assessment of the adaptive intervention occurs every two weeks until week 36. Based on whether the participant meets her GWG goal,

TABLE II: Model parameters for the simulation studies. Time constants (τ_i) are in units of days.

Parameter	EI-TPB	PA-TPB	Parameter	EI-TPB	PA-TPB
b_1	3	1	e_1	6	4
n_1	2	7	m_1	3	8
p_1	1	4	c_1	2	2
τ_1	1	30	γ_{11}	1	0.7
τ_2	1	30	γ_{22}	1	0.5
τ_3	1	10	γ_{33}	1	0.7
τ_4	1	20	β_{41}	1	0.34
τ_5	1	30	β_{42}	1	0.27
$\theta_1 \dots \theta_3$	0	0	β_{43}	1	0.13
$\theta_4 \dots \theta_6$	5	10	β_{53}	1	0.08
θ_7, θ_8	10	20	β_{54}	1	0.42
$k_{11} \dots k_{1n}$	0	0	$\theta_{11} \dots \theta_{1n}$	0	0
$k_{21} \dots k_{2n}$	0	0	$\theta_{21} \dots \theta_{2n}$	0	0
$k_{31} \dots k_{3n}$	0.0005	0.0024	$\theta_{31} \dots \theta_{3n}$	0	0
λ_r	80	100	λ_d	90	200

the dosage of the intervention components will be adapted by the decision framework operated by HMPC.

Table II summarizes the model parameters in the simulation studies, including the behavioral parameters, time constants τ_i , time delays θ_i , gains assumed for the participant, and the filter parameters respectively. All these values are hypothetical but have been selected such that the simulated responses mimic those of an actual participant.

The parameters for the HMPC are as follows: the controller horizons are $p = 40$ and $m = 25$, $Q_y = 1$, $\alpha_r = 0.2$, $\alpha_d = 0.2$, $f_a = 0.5$ and the sampling time (assessment cycle) is $T = 14$ days. The gains for the approximated energy balance model in this case study for the overweight participant are $K_{EI} = 1.785 \times 10^{-4}$, and $K_{PA} = -0.52718$.

Fig. 4 shows the simulation result for the participant's changes in maternal body mass and the energy balance variables, the intervention components dosages, as well as the PBC inflow to the EI-TPB and PA-TPB models. We can see that the participant has a ramp energy intake increase in the first trimester. During the intervention (week 14 - week 36), PBC inflows to the TPB models increase, and they almost keep constant when the intervention stops. Just before the intervention begins, the participant has already had higher energy intake than the EI reference value she should take for the third trimester suggested by IOM. Once the intervention starts, the participant accepts the base dosage (+1) for all the intervention components. I_1 is the first to be augmented, which occurs at week 16. At each assessment cycle, the augmentation has only one step up on one intervention component if necessary. So does the reduction. I_2 are not augmented until week 20, which is one assessment cycle after I_1 reaches its maximum dosage at week 18. This meets our requirement for the design of the controller. At around day 230, the participant's weight is within the upper bound of the IOM guidelines, and that is why the HMPC controller which operates the decision policy reduces the dosage of I_2 first, while still keeping I_1 at its maximum dosage. Even though when the intervention stops, the built-up PBC inflows by the intervention and self regulation still keep the participant engaging in a HE and PA active life style. At late pregnancy, the participant is able to control her energy intake at a stable level, while controlling her GWG within the IOM guideline.

V. CONCLUSIONS AND FUTURE WORK

This paper demonstrates the application of HMPC to design an adaptive behavioral intervention for GWG, based on a previously developed dynamical systems model. The problem considers hybrid dynamics in an MLD framework, and focuses on how to systematically specify the magnitude and sequence of intervention components dosages. The manner in which self-regulation is modeled in the system has been revised relative to previous work by use of the IMC technique. A case study based on an overweight participant demonstrates the functionality and benefits of the HMPC decision framework. By changing the intervention dosage magnitudes and dosage sequence relying on the model and

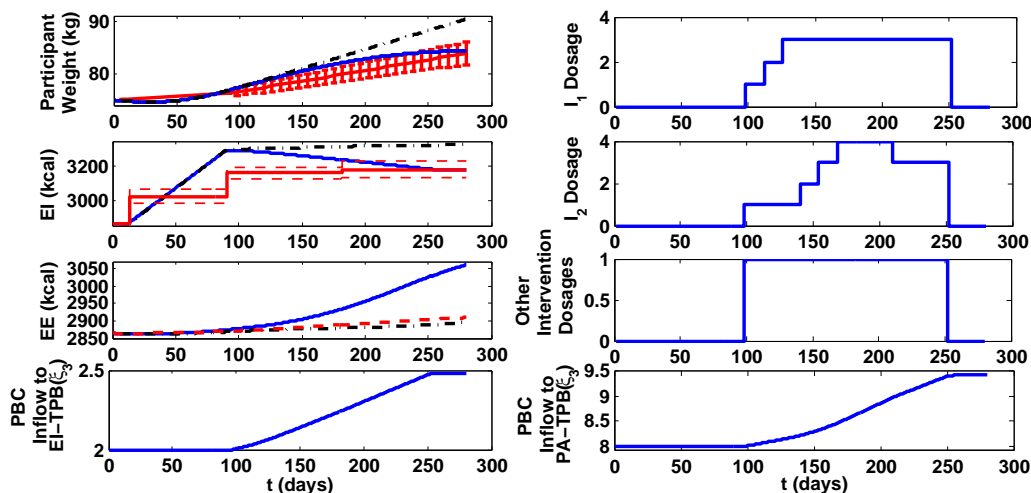


Fig. 4: Simulation responses for the maternal body mass, EI and EE, the intervention components dosages, and the PBC inflows to the two TPB models. Red lines represent the 2009 IOM guidelines applied on a daily basis; the blue solid line represent the case with intervention and self-regulation while the black dashed line represents the case with no intervention.

measured outcomes collected during treatment, the HMPC-based intervention and the participant's self-regulation work together to increase PBC, improve participant's HE and PA, and ultimately allow the participant to maintain her GWG within IOM guidelines, even in the presence of disturbances.

This work is proof of concept focusing on the application of control theory in the field of psychology and behavioral health. We are currently pursuing some interesting extensions of this research, such as specifying parameters to better understand the variations in participant behavior, and the design of clinical trials to experimentally verify both the model and the control scheme. The ultimate goal is to develop a real-life implementation of the HMPC-based intervention that is both practical and useful in behavioral settings.

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