Dynamic Modeling / Control Engineering

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Dynamical Systems and Control Engineering: How Can These Impact Behavioral Interventions?

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About Daniel E. Rivera

Education:

- B.S. ChE degree from the Univ. of Rochester (1982)
- M.S. ChE degree from the Univ. of Wisconsin (1984)
- Ph.D. from California Institute of Technology (1987)

Positions Held:

- Associate Research Engineer, Shell Development Co., (1987 - 1990)
- Professor of Chemical Engineering, Arizona State University (1990 - present)

Other Professional Activities:

- Senior Member, AIChE and IEEE
- Chair, IEEE-Control Systems Society, Technical Committee on System Identification and Adaptive Control (TC-SIAC)

http://csel.asu.edu/health
Current Projects in Behavioral Health

- R21DA024266*, “Dynamical systems and related engineering approaches to improving behavioral interventions,” NIH Roadmap Initiative Award on Facilitating Interdisciplinary Research Via Methodological and Technological Innovation in the Behavioral and Social Sciences, with L.M. Collins, Penn State, co-PI.

- K25DA021173*, “Control engineering approaches to adaptive interventions for fighting drug abuse,” Mentors: L.M. Collins (Penn State) and S.A. Murphy (Michigan).

*Projects funded by the US National Institutes of Health: NIDA (National Institute on Drug Abuse) and OBSSR (Office of Behavioral and Social Sciences Research).

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Current Research Activities

- Dynamical systems modeling, system identification, and control engineering frameworks for delivering optimized time-varying adaptive interventions.

- Analysis of smoking activity and cessation as closed-loop dynamical systems (M. Piper, T. Baker, and M. Fiore, U of Wisconsin-Center for Tobacco Research and Intervention).

- Dynamic modeling and optimization of a preventive intervention for excessive weight gain during pregnancy (D. Downs and J. Savage, Penn State University; D. Thomas, Montclair State University).

- Dynamic modeling of diary data from a low-dose naltrexone intervention in fibromyalgia patients; feasibility of system identification modeling and Model Predictive Control for pain treatment interventions (J. Younger, Stanford University School of Medicine).
About this lecture

- Goal is to discuss how dynamical systems and engineering control theory can improve the design and implementation of time-varying adaptive interventions, and illustrate this in various application settings.

- Talk will be focused on describing important concepts; it will not be a comprehensive survey.

- We will attempt to establish connections between behavioral health, quantitative methods in the behavioral and social sciences, and engineering, discussing the opportunities (and challenges) that these present to the behavioral scientist, the methodologist, and the engineer.

Lecture Outline

- Fundamentals of adaptive interventions
- What is meant by control systems engineering, and how can these concepts improve behavioral interventions?
  - Shower operation as a closed-loop dynamical system.
  - Analysis and design of a hypothetical time-varying adaptive intervention inspired by the Fast Track program.
- Some additional illustrations
  - Fibromyalgia intervention using low-dose naltrexone.
  - Dynamic modeling of a weight loss intervention.
  - Mediation modeling of smoking cessation.
Behavioral Interventions

- Behavioral interventions aim to prevent and treat disease by reducing unhealthful behaviors and promoting healthful ones.

- These play an increasingly prominent role in a wide variety of areas of public health importance, among them drug and alcohol abuse, cancer, mental health, obesity, HIV/AIDS, and cardiovascular health.

- Interventions can include components that are either pharmacological (e.g., naltrexone, bupropion) or behavioral (e.g., motivational interviewing, cognitive behavioral therapy, relaxation exercises) in nature. Likewise, these interventions can be designed to address multiple outcomes (e.g., co-morbidities).

- Adaptive interventions (in contrast to fixed interventions) represent an important emerging paradigm for delivering behavioral interventions intended to address chronic, relapsing disorders (Collins, Murphy, and Bierman, Prevention Science, 5, No. 3, 2004).

Basic Components of Adaptive Interventions
(Collins, Murphy, and Bierman, Prevention Science, 5, No. 3, 2004)

- The assignment of a particular dosage and/or type of treatment is based on the individual’s values on variables that are expected to moderate the effect of the treatment component; these are known as tailoring variables.

- In a time-varying adaptive intervention, the tailoring variable is assessed periodically, so the intervention is adjusted on an on-going basis.

- Decision rules translate current and previous values of tailoring variables into choice(s) of treatment and their appropriate dosage.
Adaptive Intervention Benefits
(Collins, Murphy, and Bierman, *Prevention Science*, 5, No. 3, 2004)

- An effective adaptive intervention strategy may result in the following advantages over fixed interventions:
  - Reduction of negative effects (i.e., stigma),
  - Reduction of inefficiency and waste,
  - Increased compliance,
  - Enhanced intervention potency.

- Adaptive interventions can serve as an aid for disseminating efficacious interventions in real-world settings.

Adaptive Intervention Simulation
(inspired by the *Fast Track Program*, Conduct Problems Prevention Research Group)

- A multi-year program designed to prevent conduct disorder in at-risk children.

- Frequency of home-based counseling visits assigned quarterly to families over a three-year period, based on an assessed level of parental functioning.

- Parental function (the tailoring variable) is used to determine the frequency of home visits (the intervention dosage) according to the following decision rules:
  - If parental function is “very poor” then the intervention dosage should correspond to weekly home visits,
  - If parental function is “poor” then the intervention dosage should correspond to bi-weekly home visits,
  - If parental function is “below threshold” then the intervention dosage should correspond to monthly home visits,
  - If parental function is “at threshold” then the intervention dosage should correspond to no home visits.
• The assigned dosage (frequency of counseling visits) decreases as the value of the tailoring variable (parental function) increases, as prescribed by the decision rules.

Single participant family scenario. Offset (where parental function fails to reach a desired goal at the end of the intervention) occurs in this case.
Control Systems Engineering

The field that relies on dynamical models to develop mechanisms for adjusting system variables so that their behavior over time is transformed from undesirable to desirable,

• **Open-loop**: refers to system behavior without a controller or decision rules (i.e., MANUAL operation).

• **Closed-loop**: refers to system behavior once a controller or decision rule is implemented (i.e., AUTOmatic operation).

A well-tuned control system will effectively transfer variability from an expensive system resource to a less expensive one.

• The field that relies on dynamical models to develop algorithms for adjusting system variables so that their behavior over time is transformed from undesirable to desirable.

• Control engineering plays an important part in many everyday life activities. Some examples of control systems engineering:
  - Cruise control and climate control in automobiles,
  - The “sensor reheat” feature in microwave ovens,
  - Home heating and cooling,
  - The insulin pump for Type-I diabetics,
  - “Fly-by-wire” systems in high-performance aircraft,
  - Homeostasis

• A well-tuned control system will effectively transfer variability from the more “expensive” system resource to a less expensive one.
Control Systems Engineering Laboratory

Open-Loop (Manual) vs. Closed-Loop (Automatic) Control

- Climate control in automobiles is one of many illustrations of closed-loop control that can be found in daily life.

The “Shower” Problem

Controlled variables ($y$): Temperature, water flow

Manipulated Variables ($u$): Hot and Cold Water Valve Positions

Disturbances ($d$): Inlet Water Flows, Temperatures

Objective: Adjust hot and cold water flows in response to changes in shower temperature and outlet flow caused by external factors.
Controlled Variables \((y; \text{outcomes})\): system variables that we wish to keep at a reference value (or goal), also known as the setpoint \((r)\).

Manipulated Variables \((u)\): system variables whose adjustment influences the response of the controlled variable; their value is determined by the controller/decision policy.

Disturbance Variables \((d)\): system variables that influence the controlled variable response, but cannot be manipulated by the controller; disturbance changes are external to the system.

Both manipulated \((u)\) and disturbance \((d)\) variables can be viewed as independent \((x)\) variables; disturbances are exogenous, while manipulated variables can be adjusted by the user.

Consider the change in shower temperature caused by a sudden drop in inlet water flowrate as a result of a disturbance (e.g., sprinklers being activated).
Sensors (i.e., assessment instruments): devices needed to measure the controlled and (possibly) the disturbance variables.

Actuators: devices needed to achieve desired settings for the manipulated variables.

Controllers (i.e., clinical decision rules). These relate current and prior controlled variable, manipulated variable, and disturbance measurements to a current value for the manipulated variable.

In feedback control:

- the measured controlled variable \((y)\) is compared to a goal (also known as a reference setpoint \(r\)),

- a control error \(e (= r - y)\), representing the discrepancy between \(y\) and \(r\) is calculated.

- a control algorithm determines a current value for the manipulated variable \((u)\) based on current and previous values of \(e\) and \(u\).
The “Magic” of Feedback
(Adapted from K. J. Åström’s “Challenges in Control Education” plenary talk at the 7th IFAC Symposium on Advances in Control Education, Madrid, Spain, June 21-23, 2006).

Feedback has some amazing properties:

- can create good systems from bad components,
- makes a system less sensitive to disturbances and component variations,
- can stabilize an unstable system,
- can create desired behavior, for example, linear behavior from nonlinear components.

Major drawback: it can cause instability if not properly tuned.

Shower Problem: Closed-Loop Feedback Control
(Good: Stable, Smooth Responses)
Shower Problem: Closed-Loop Feedback Control
(Bad: Unstable, Oscillatory Responses)

Controller

Sensor

Actuator

Hot

Cold

Temp. setpoint

Disturbance: Inlet Water Flow

Manipulated: Hot Water Valve Position

Controlled: Temperature

Open-Loop
(Before Control)

Closed-Loop Control

The transfer of variance from an expensive resource to a cheaper one is one of the major benefits of control systems engineering.
Adaptive Intervention Simulation
(inspired by the Fast Track Program, Conduct Problems Prevention Research Group)

• A multi-year program designed to prevent conduct disorder in at-risk children.

• Frequency of home-based counseling visits assigned quarterly to families over a three-year period, based on an assessed level of parental functioning.

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  - If parental function is “at threshold” then the intervention dosage should correspond to no home visits.

Parental Function Feedback Loop Block Diagram*
(to decide on home visits for families with at-risk children)

Parental Function - Home Visits Adaptive Intervention as an Inventory Control Problem

\[ PF(t + 1) = PF(t) + K_I I(t - \theta) - D(t) \]

Parental function \(PF(t)\) is built up by providing an intervention \(I(t)\) (frequency of home visits), that is potentially subject to delay, and is depleted by potentially multiple disturbances (adding up to \(D(t)\)).

Parental Function “Open Loop” Dynamics

\[ PF(t+1) = PF(t) + K_I I(t - \theta) - D(t) \]

\(t = \text{time, expressed as an integer reflecting review instance}\)
\(K_I = \text{intervention gain}\)
\(D(t) = \text{depletion}\)
\(I(t) = \text{intervention dosage}\)
\(\theta = \text{delay time}\)

Parental Function (End of Review Instance)

\[ = \text{Parental Function (Start of Review Instance)} \]
\[ + \text{Parental Function Contributed by Intervention} \]
\[ - \text{Parental Function Depletion} \]
Parental Function Dynamics
“Open Loop” Response

- Variations in intervention dose response for a single participant family

- Between-participant variability as a result of individual dynamic characteristics
• Parental function change as a result of step changes in outflow (the disturbance variable) of varying magnitudes.

Adaptive Intervention Using “IF-THEN” Rules

No Depletion ($D(t) = 0$)  High Depletion ($D(t) = 5$)

Single participant family scenario. The goal is for the family to attain a 50% proficiency (dashed line) on a parental function scale at the conclusion of the three year intervention. Offset (where parental function fails to meet goal) is more pronounced when high depletion is present.
Adaptive Intervention Using “IF-THEN” Rules

Multiple participant family simulation. The goal is for each family to attain a 50% proficiency (dashed line) on a parental function scale at the conclusion of the three year intervention. Offset is observed in all participant families.

- Based on a knowledge of the open-loop model, an optimized feedback decision algorithm (i.e., the “controller”) can be designed for this system.

- In general, the sophistication of the controller will be a function of the complexity of the model and the desired performance requirements.

- We consider a tuning rule for a Proportional-Integral Derivative (PID) feedback controller for an “integrating” system which relies on the concept of Internal Model Control (IMC; Morari and Zafiriou, 1987).

- User supplies the intervention gain ($K_i$) and delay ($\theta$) and a setting for an adjustable parameter ($\lambda$) that defines the speed of response and robustness of the control system.
Control Design Requirements

- Stability. Many different notions exist, but “BIBO” stability (bounded inputs resulting in bounded outputs) is usually sufficient.
- No offset. Control error \( e = r - y \) should go to zero (meaning that the controlled variable should reach the goal) at a finite time.
- Minimal effect of disturbances on controlled variables.
- Rapid, smooth (i.e., non-oscillatory) responses of controlled variables to setpoint changes.
- Large variations (“moves”) in the manipulated variables should be avoided.
- Robustness, that is, performance should display little sensitivity to changes in operating conditions and model parameters.

Proportional-Integral-Derivative (PID) with Filter Controller Summary

\[ l(t) = l(t-1) + K_1 e(t) + K_2 e(t-1) + K_3 e(t-2) + K_4 (l(t-1)-l(t-2)) \]

Current Dosage = Previous Dosage + Scaled Corrections using Current and Prior Control Errors + Scaled Previous Dosage Change

- \( K_1, K_2, K_3, \) and \( K_4 \) are tuning constants in the controller;
- \( e(t) = (PF(t) - PF_{Goal}) \), where \( PF_{Goal} \) is the setpoint (“goal”) and \( e(t) \) is the control error.
- The dosage decision \( l(t) \) is a continuous value between 0 and 100%, but for purposes of this example it is quantized into the nearest of the four dosage levels (\( l_{weekly}, l_{biweekly}, l_{monthly}, 0 \)).
Internal Model Control-Proportional Integral Derivative (IMC-PID)
Controller Tuning Rules (Rivera et al., 1986)

User supplies open-loop model gain ($K_i$), delay ($\theta$) and the adjustable parameter ($\lambda$); $T$ is the review period

$$I(t) = I(t-T) + K_1e(t) + K_2e(t-T) + K_3e(t-2T) + K_4(I(t-T) - I(t-2T))$$

$$\beta = \tau = \frac{\theta}{2}$$

$$K_c = \frac{2(\beta+\lambda)+\tau}{K_1(2\beta^2+4\beta\lambda+\lambda^2)} \quad \tau_I = 2(\beta + \lambda) + \tau \quad \tau_D = \frac{2\tau(\beta+\lambda)}{2(\beta+\lambda)+\tau} \quad \tau_F = \frac{\beta\lambda^2}{2\beta^2+4\beta\lambda+\lambda^2}$$

$$K_1 = \frac{T K_c}{\tau_F + T} \left(1 + \frac{T}{\tau_I} + \frac{\tau_D}{T}\right) \quad K_2 = -\frac{T K_c}{\tau_F + T} \left(1 + \frac{2\tau_D}{T}\right) \quad K_3 = \frac{K_c \tau_D}{\tau_F + T} \quad K_4 = \frac{\tau_F}{\tau_F + T}$$

Controller/Decision Rule Comparison, High Depletion Rate ($D(t) = 5$)

“IF-THEN” rules  
IMC-PID control ($\lambda = 3$; moderate speed)

36 month intervention reviewed at quarterly intervals. Offset problem is eliminated by more judicious assignment of intervention dosages during the course of the intervention.
IMC-PID Controller, High Depletion Rate, Various Controller Speeds (determined by λ)

Fast (λ = 1)

Moderate (λ = 3)

Slow (λ = 5)

IF-THEN vs. IMC-PID Comparison

The intervention dosage is adapted at quarterly intervals over a 36-month time period. The goal is for each family to attain a 50% proficiency (dashed line) on a parental function scale at the conclusion of the three year intervention.

“IF-THEN” Decision Rules

IMC-PID control (λ = 3)

• The closed-loop response of five participant families is evaluated using a controller model based on an average (“nominal”) effect. The “transfer of variance” concept is illustrated here.
The intervention dosage is adapted at quarterly intervals over a 36-month time period. The goal is for each family to attain a 50% proficiency (dashed line) on a parental function scale at the conclusion of the three year intervention.

**IMC-PID control ($\lambda = 1$)**

**IMC-PID control ($\lambda = 5$)**

- The closed-loop response of five participant families is evaluated using a controller model based on an average ("nominal") effect. The “transfer of variance” concept is illustrated here.

### Additional Topics

- **Feedforward control action**: if disturbance variables can be measured, these can be incorporated as tailoring variables in the controller in a feedforward (i.e., anticipative) manner.

- **Model predictive control**: control design paradigm that features advanced adaptive functionality such as constraint handling, decision-making involving multiple outcomes, and formal assignment of intervention dosages to discrete-valued categories.

- **System identification**: examines how to empirically obtain dynamical models from data; also enables simplifying other model types (e.g., system dynamics, agent-based models) into forms amenable for control.
Fibromyalgia Intervention Study


• Fibromyalgia (FM) is a condition characterized by chronic pain; its etiology is not well understood. Daily report data for a representative participant in a pilot study using low-dose naltrexone (LDN) as treatment for fibromyalgia is shown above.


Dynamic Modeling Challenges

• Limited a priori knowledge regarding the dynamics of the system.

• Secondary data analysis on fixed protocol that was not designed for dynamical system analysis.

• No convenient means to generate a cross-validation dataset.

• Determinations of system input, outputs, and disturbances somewhat arbitrary
  - Outputs (y): Typical symptom indicators such as FM symptoms, overall sleep, highest pain
  - Inputs (u,d): LDN and placebo as primary inputs (u); exogeneous effects such as anxiety or stress as disturbances (d).
Data Preprocessing: The data is preprocessed for missing data entries and is smoothened using a three day moving average.

Discrete-time parametric modeling: The filtered data is fitted to a AutoRegressive with eXternal input (ARX-[na nb nk]) parametric model:

$$\begin{align*}
y(t) + \ldots + a_{n_a} y(t - n_a) &= b_{11} u_1(t - n_k) + \ldots + b_{n_b 1} u_1(t - n_k - n_b + 1) \\
& \quad + b_{1i} u_i(t - n_k) + \ldots + b_{n_b i} u_i(t - n_k - n_b + 1) \\
& \quad \vdots \\
& \quad + b_{1n_u} u_{n_u}(t - n_k) + \ldots + b_{n_b n_u} u_{n_u}(t - n_k - n_b + 1) + e(t)
\end{align*}$$

Simplification to a continuous time model: The step responses from the ARX model are individually fit to a parsimonious continuous-time dynamical model of the form:

$$\tau^2 \frac{d^2 y}{dt^2} + 2\zeta \tau \frac{dy}{dt} + y(t) = K_p \left( u(t) + \tau_a \frac{du}{dt} \right)$$

Multi Input Model Construction

The multi-input ARX-[2 2 1] models (with respective input(s)) are:

- Model 1 (Drug)
- Model 2 (Drug, Placebo)
- Model 3 (Drug, Placebo, Anxiety)
- Model 4 (Drug, Placebo, Anxiety, Stress)
- Model 5 (Drug, Placebo, Anxiety, Stress, Mood)
\[ \tau^2 \frac{d^2 y}{dt^2} + 2\zeta \tau \frac{dy}{dt} + y(t) = K_p \left( u(t) + \tau_a \frac{du}{dt} \right) \]

<table>
<thead>
<tr>
<th>Model (Input-Output)</th>
<th>( K_p, \tau, \zeta, \tau_a )</th>
<th>( T_{98%}(\text{days}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drug-FM symptoms</td>
<td>-2.47, 1.57, 1.26, 1.96</td>
<td>11.49</td>
</tr>
<tr>
<td>Placebo-FM symptoms</td>
<td>45.81, 1.57, 1.26, 1.15</td>
<td>13.06</td>
</tr>
<tr>
<td>Anxiety-FM symptoms</td>
<td>0.86, 1.57, 1.26, 0.24</td>
<td>14.24</td>
</tr>
<tr>
<td>Stress-FM symptoms</td>
<td>2.29, 1.57, 1.26, 0.49</td>
<td>13.94</td>
</tr>
<tr>
<td>Mood-FM symptoms</td>
<td>-0.091, 1.57, 1.26, 4.67</td>
<td>11.93</td>
</tr>
<tr>
<td>Drug-Overall Sleep</td>
<td>4.98, 2.13, 1.04, -3.35</td>
<td>15.83</td>
</tr>
</tbody>
</table>


\[ \tau^2 \frac{d^2 y}{dt^2} + 2\zeta \tau \frac{dy}{dt} + y(t) = K_p \left( u(t) + \tau_a \frac{du}{dt} \right) \]

- Rise time, settling time, overshoot, oscillation, and inverse response are important characteristics of this model response.
Model Predictive Control (MPC)

- Control engineering technology widely used in many industrial applications (from chemical mfg to automotive and aerospace).
- As an optimization technology, MPC can minimize (or maximize) an objective function that represents a suitable metric of intervention performance.
- As a control system, MPC accomplishes feedback (and feedforward action) in the presence of model error, measurement unreliability, and disturbances that may affect the intervention.
- Three major steps in MPC:
  - Prediction of intervention outcomes at time instants in the future (i.e., the prediction horizon) based on a model,
  - Optimization of a sequence of future dosage decisions through minimizing (or maximizing) an objective function,
  - Receding horizon strategy.

Model Predictive Control Conceptual Representation

\[
\begin{align*}
\min \left\{ (u(k+i))_{j=0}^{m-1}, [\delta(k+i)]_{i=0}^{P-1}, [z(k+i)]_{i=0}^{P-1} \right\} \\
J \triangleq \sum_{i=1}^{P} \| (y(k+i) - y_r) \|^2_{Q_y}
\end{align*}
\]
Model Predictive Control Optimization Problem

\[
\begin{align*}
\min_{\{u(k+i)\}_{i=0}^{m-1}, \{\delta(k+i)\}_{i=0}^{p-1}, \{z(k+i)\}_{i=0}^{p-1}} & J \triangleq \sum_{i=1}^{p} \| (y(k+i) - y_r) \|_{Q_y}^2 \\
\end{align*}
\]

\[y_{\min} \leq y(k+i) \leq y_{\max}, \quad 1 \leq i \leq p \]

\[u_{\min} \leq u(k+i) \leq u_{\max}, \quad 0 \leq i \leq m - 1 \]

\[\Delta u_{\min} \leq \Delta u(k+i) \leq \Delta u_{\max}, \quad 0 \leq i \leq m - 1 \]

Take controlled variables (primary outcomes) to goal, subject to restrictions on:

- manipulated variable range limits (i.e., intervention dosage limits)
- the rate of change of manipulated variables (i.e., dosage changes)
- controlled variables (i.e., primary outcome variables),
- associated variables (i.e., secondary outcomes)

Closed-Loop Control, Naltrexone Intervention for Fibromyalgia


• Simulation result showing MPC system response to goal change (stpt. tracking), increase in anxiety report (measured disturbance rejection) and unpredicted change in pain report (unmeasured disturbance rejection).
The dynamical model consists of a system of integrated differential equations describing:

- **Physiology (energy balance),**
- **Behavior change (Theory of Planned Behavior).**


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**Physiological Model:**

- **Energy balance**

*Three compartment model based on work of K. Hall and C. Chow, NIDDK intramural scientists*

- **Inputs:**
  - Diet
    - Carbohydrates (CI)
    - Fat (FI)
    - Proteins (PI)
    - Sodium (Na)
  - Physical Activity (PA; \( \delta \), in kcal/kg)

- **Outputs:**
  - Fat-Free Mass:
    - Lean Tissue Mass (LTM)
    - Extracellular Fluid (ECF)
  - Fat Mass (FM)
Model proposed by K. Hall and C. Chow (NIDDK Intramural Scientists). Daily energy balance $EB(t)$:

$$EB(t) = EI(t) - EE(t),$$

$EI(t)$ energy intake and $EE(t)$ energy expenditure, both in kcal/d.

- **Energy Intake $EI$:**

  $$EI(t) = a_1 CI(t) + a_2 FI(t) + a_3 PI(t)$$

  $CI, FI, PI$ carbohydrate, fat, and protein intakes all in grams/day, $a_1 = 4$ kcal/gram, $a_2 = 9$ kcal/gram, and $a_3 = 4$ kcal/gram.

- **Energy Expenditure $EE$:**

  $$EE(t) = \beta EI(t) + \delta BM + K + \gamma_{LTM} LTM(t) + \gamma_{FM} FM(t) +$$
  $$\eta_{FM} \frac{d FM}{dt} + \eta_{LTM} \frac{d LTM}{dt}$$

  $\delta$ is the physical activity coefficient (in kcal/kg), $LTM$ lean tissue mass and $FM$ fat mass. $K$ is a steady-state constant.

Three-compartment model ($FM, LTM$ and extra-cellular fluid $ECF$):

$$\frac{d FM}{dt} = \frac{(1 - p(t))EB(t)}{\rho_{FM}}$$

$$\frac{d LTM}{dt} = \frac{p(t)EB(t)}{\rho_{LTM}}$$

$$\frac{d ECF}{dt} = \Delta Na_{diet} - \xi_{Na}(ECF - ECF_{init}) - \xi_{CI}(1 - CI/CI_b) \frac{[Na]}{\tau_{Na}},$$

$p$ is given by the Forbes formula:

$$p = \frac{C}{(C + FM)}; \quad C = 10.4 \frac{\rho_{LTM}}{\rho_{FM}}.$$  

For the extracellular fluid volume (in ml), $\Delta Na_{diet}$ is the change on sodium in mg/d, $CI_b$ is the baseline carbohydrate intake, $ECF_{init}$ is the initial ECF volume and $\tau_{Na} = 2$ is a time constant of two days.

Finally, for the body mass:

$$BM(t) = FM(t) + LTM(t) + ECF(t).$$
Interactive tool that enables evaluating the three-compartment energy balance model; can be downloaded from http://csel.asu.edu/Weigh-IT

**Psychological model: Theory of Planned Behavior (TPB)**

Theory of Planned Behavior (TPB, Ajzen, 1985) says that behavior is influenced by intention, which in turn is influenced by:

- **Attitude toward the behavior**: determined by the **strength of beliefs about the outcome** \( b \) and the **evaluation of the outcome** \( e \).
  
- **Subjective norm(s)**: determined by **normative beliefs** \( n \) and the **motivation to comply** \( m \).
  
- **Perceived Behavioral Control**: determined by the **strength of each control belief** \( c \) and the **perceived power of the control factor** \( p \).
\[ \eta = B \eta + \Gamma \xi + \zeta \]

\[
\begin{bmatrix}
\eta_1 \\
\eta_2 \\
\eta_3 \\
\eta_4 \\
\eta_5
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\beta_{41} & \beta_{42} & \beta_{43} & 0 & 0 \\
0 & 0 & \beta_{53} & \beta_{54} & 0
\end{bmatrix}
\begin{bmatrix}
\eta_1 \\
\eta_2 \\
\eta_3 \\
\eta_4 \\
\eta_5
\end{bmatrix} +
\begin{bmatrix}
\gamma_{11} & 0 & 0 & 0 \\
0 & \gamma_{22} & 0 & 0 & 0 \\
0 & 0 & \gamma_{33} & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\xi_1 \\
\xi_2 \\
\xi_3 \\
\xi_4 \\
\xi_5
\end{bmatrix} +
\begin{bmatrix}
\zeta_1 \\
\zeta_2 \\
\zeta_3 \\
\zeta_4 \\
\zeta_5
\end{bmatrix}
\]

Fluid Analogy for the Theory of Planned Behavior
The conservation principle (Accumulation = Inflow – Outflow) leads to the following system of differential equations:

\[
\begin{align*}
\tau_1 \frac{d\eta_1}{dt} &= \gamma_{11} \xi_1(t) - \eta_1(t) + \zeta_1(t) \\
\tau_2 \frac{d\eta_2}{dt} &= \gamma_{22} \xi_2(t) - \eta_2(t) + \zeta_2(t) \\
\tau_3 \frac{d\eta_3}{dt} &= \gamma_{33} \xi_3(t) - \eta_3(t) + \zeta_3(t) \\
\tau_4 \frac{d\eta_4}{dt} &= \beta_{41} \eta_1(t) + \beta_{42} \eta_2(t) + \beta_{43} \eta_3(t) - \eta_4(t) + \zeta_4(t) \\
\tau_5 \frac{d\eta_5}{dt} &= \beta_{54} \eta_4(t) + \beta_{53} \eta_3(t) - \eta_5(t) + \zeta_5(t),
\end{align*}
\]

where:
\(\tau_1, \cdots, \tau_5\) are time constants,
\(\eta_1, \cdots, \eta_5\) are the inventories,
\(\xi_1(t) = b_1(t) e_1(t), \xi_2(t) = n_1(t) m_1(t), \xi_3(t) = c_1(t) p_1(t),\)
\(\gamma_{11}, \cdots, \gamma_{33}\) are the inflow resistances,
\(\beta_{41}, \cdots, \beta_{54}\) are the outflow resistances,
\(\theta_1, \cdots, \theta_7\) are time delays and \(\zeta_1, \cdots, \zeta_5\) are disturbances.

System response for various step change increases in strength of beliefs to attitude:

\(\xi_1 = b_1 \times e_1: \)
(solid) \(b_1 = 4, e_1 = 4 \)
(dashed) \(b_1 = 6, e_1 = 6 \)
(dot-dashed) \(b_1 = 8, e_1 = 8 \)

Model parameters:
\(\theta_1 = \cdots \theta_3 = 0, \theta_4 = \cdots \theta_7 = 2\)
\(\tau_1 = \cdots \tau_3 = 1, \tau_4 = 2, \tau_5 = 4\)
\(\gamma_{ij} = 1, \beta_{ij} = 0.5, \zeta_i = 0\)
• Two simulation examples that illustrate the use of the integrated model are described in:


• Simulation Example I: Using the model to better understand variability in participant response;

• Simulation Example II: Using the model to better understand the proper sequence of intervention components;

Representative male participant with initial conditions: $BM = 100$ kg, $FM = 30$ kg, $LTM = 45$ kg and $ECF = 25$ liters.

*Intervention acts upon:* beliefs about healthy eating habits (from $b_1 = 7$ to $b_1 = 10$) and healthy physical activity (from $b_1 = 1$ to $b_1 = 3$).

**Three Scenarios:**

i) Complete assimilation, $\tau_1 = 0.1, \beta_{41} = 1, \theta_7 = 0$.

ii) Partial assimilation, $\tau_1 = 20, \beta_{41} = 0.5, \theta_7 = 15$.

iii) Noisy partial assimilation, $\zeta_1 \sim N(0, 20)$ for EI and $\zeta_1 \sim N(0, 50)$ for PA.

**Additional parameters:**

• Behavioral variables: $\xi_2 = \xi_3 = 50$,

• Delays: $\theta_1 = \cdots = \theta_6 = 0, \theta_8 = 0$,

• Time constants: $\tau_2 = \cdots = \tau_5 = 0.1$,

• Inflow and outflow resistances: $\gamma_{ij} = 1$ and $\beta_{ij} = 0.5$. 
Behavioral Response

Integrated Model, Simulation Example: Understanding Participant Variability
• Excessive Gestational Weight Gain (GWG) increases risk factors for pregnancy complications such as gestational diabetes, macrosomia, preeclampsia, and birth defects.

• Over the past 20 years, the percentage of women gaining over 40 lbs (18 kg) during pregnancy has increased by 30%.

• Our approach relies on dynamical systems modeling to predict weight change during pregnancy, incorporating both physiological and psychological factors:
  - Physiological model: maternal-fetal energy balance model.
  - Psychological model: mechanistic model inspired by the Theory of Planned Behavior (TPB).

---

Conceptual Model for Gestational Weight Gain Interventions
• Data from study described in McCarthy et al., *Addiction*, Vol. 103, pgs. 1521-1533, 2008. Active drug is buproprion SR.

• 11 week study; randomization (n = 463)
  - Drug: Drug, Placebo
  - Counseling: Yes, No

• Treatment Conditions:
  - Active Drug with Counseling (AC; n=101)
  - Active Drug, No Counseling (ANc; n = 101)
  - Placebo with Counseling (PC; n =100)
  - Placebo, No Counseling (PNc ; n =101)

• T = 42 daily observations for each participant

Fluid Analogy for Mediation Analysis

Path Diagram:

\[ \begin{align*}
\tau_1 \frac{dM}{dt} &= a \left( T(t - \theta_1) - M(t) + e_1(t) \right) \\
\tau_2 \frac{dY}{dt} &= c' \left( T(t - \theta_2) + b \left( M(t - \theta_3) - Y(t) \right) + e_2(t) \right).
\end{align*} \]

Time constant (τ) and delay (θ) variables are essential features in this dynamic model representation for mediation.
• Comparison of average cigarettes smoked and craving scores for two treatment groups (active drug with counseling (blue) vs. placebo-no counseling (red)).

• Participant “A” from drug group (blue); participant “B” from placebo group (red)
Parameter Estimation Summary

\[
\begin{align*}
\tau_1 \tau_2 \frac{d^2 M}{dt^2} + (\tau_1 + \tau_2) \frac{dM}{dt} + M(t) &= a(T(t) + \tau_a \frac{dT}{dt}) \\
\tau_3 \tau_4 \frac{d^2 Y}{dt^2} + (\tau_3 + \tau_4) \frac{dY}{dt} + Y(t) &= c'(T(t) + \tau_3 \frac{dT}{dt}) + b(M(t) + \tau_4 \frac{dM}{dt})
\end{align*}
\]

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Cohort Average, Active Drug with Counseling</th>
<th>Cohort Average, Placebo with No Counseling</th>
<th>Participant A, Active Drug with Counseling</th>
<th>Participant B, Placebo with No Counseling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mediator Fit [%]</td>
<td>86.12</td>
<td>63.74</td>
<td>65.70</td>
<td>47.70</td>
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<tr>
<td>Outcome Fit [%]</td>
<td>84.85</td>
<td>88.05</td>
<td>74.02</td>
<td>58.88</td>
</tr>
<tr>
<td>(a)</td>
<td>-11.088</td>
<td>-3.779</td>
<td>-18.539</td>
<td>2.611</td>
</tr>
<tr>
<td>(\tau_1)</td>
<td>7.669</td>
<td>17.090</td>
<td>6.099</td>
<td>1.722</td>
</tr>
<tr>
<td>(\tau_2)</td>
<td>0.281</td>
<td>0.001</td>
<td>0.001</td>
<td>13.424</td>
</tr>
<tr>
<td>(\tau_3)</td>
<td>-3.275</td>
<td>-28.014</td>
<td>-7.314</td>
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</tr>
<tr>
<td>(\tau_4)</td>
<td>-15.873</td>
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<tr>
<td>(c')</td>
<td>0.507</td>
<td>0.001</td>
<td>0.188</td>
<td>9.511</td>
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<tr>
<td>(b)</td>
<td>-0.087</td>
<td>-0.286</td>
<td>0.004</td>
<td>-0.762</td>
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<tr>
<td>(\tau_f)</td>
<td>0.782</td>
<td>0.977</td>
<td>0.001</td>
<td>0.006</td>
</tr>
</tbody>
</table>

- Parameter estimation performed using the Process Models feature in Matlab's System Identification Toolbox (one-step ahead prediction-error minimization for continuous differential equation structures).
Summary and Conclusions

• Behavioral interventions, when modeled as dynamical systems, will benefit from a control engineering perspective.

• Applying a dynamical systems approach will require intensive measurement and a recognition of the input/output nature of phenomena associated with behavioral interventions.

• Connections between behavioral theory (represented via path diagrams) and dynamical systems can be established using fluid analogies. This was demonstrated for the Theory of Planned Behavior and statistical mediation.

• A hypothetical adaptive intervention based on Fast Track has been simulated using a rule-based controller ("IF-THEN" decision rules) vs. engineering-based PID (Proportional-Integral-Derivative) decision algorithms.

• System identification and model predictive control offers significant advantages as a means for implementing adaptive, time-varying behavioral interventions.
Upcoming Courses

• ChE 561 Advanced Process Control: to be offered spring 2012

• ChE 461/598 Introduction to Process Dynamics and Control: to be offered fall 2012

• ChE 598 Introduction to System Identification: to be offered spring 2013.

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http://cse1.asu.edu/health
References

- Behavioral interventions as dynamical systems; connections to path diagrams and use of system identification techniques:

- Great introductory paper on adaptive interventions:

References (Continued)

- Control engineering for adaptive interventions:
• Some tutorial presentations that may be of interest:
  
  • Rivera, D.E., “Engineering control theory: can it impact adaptive interventions?”
    tutorial presentation at 2010 Society for Prevention Research workshop, June 1, 2010.
    Can be downloaded from http://csel.asu.edu/adaptiveintervention (select item 9).
  
  • Rivera, D.E., “A brief introduction to system identification,”
    Penn State Methodology Center Brown Bag presentation, March 20, 2008.
    Can be downloaded from http://csel.asu.edu/controleducation (select item 10).
  
  • Rivera, D.E., “An introduction to mechanistic models and control theory,”
    tutorial presentation at the SAMSI Summer 2007 Program on Challenges in Dynamic Treatment
    Can be downloaded from http://csel.asu.edu/controleducation (select item 9).

• My first paper in the control engineering area:
  
  • Rivera D.E., M. Morari, and S. Skogestad, “Internal model control: PID controller design,”

• Sunil Deshpande’s MS thesis:
  
  • “A control engineering approach for designing an optimized treatment plan for